Table 3 Shear, Moment, Slope and Deflection Formulas for Elastic Straight Beams

Case 1d Concentrated Intermediate Load; Left End Fixed, Right End Fixed

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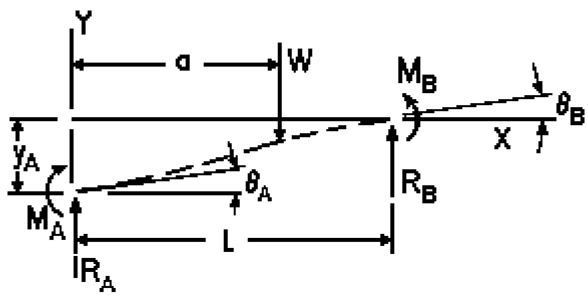
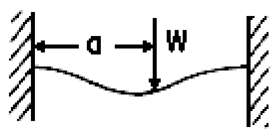
Concentrated intermediate load



Left end fixed, right end fixed



Notation file — Provides a description of Table 3 and the notation used.

Enter dimensions, properties and loading

Before progressing further, calculate the moment of inertia (I) for your cross section by flipping to Table 1. Enter the computed value below:

Table 1

Area moment of inertia: $I \equiv 7.235 \cdot in^4$

Length of beam: $L \equiv 120 \cdot in$

Distance from left edge to load: $a \equiv 60 \cdot in$

Modulus of elasticity: $E \equiv 29.5 \cdot 10^6 \cdot \frac{lbf}{in^2}$

Load: $W \equiv 1000 \cdot lbf$

Boundary values

The following specify the reaction forces (R), moments (M), slopes (θ) and deflections (y) at the left and right ends of the beam (denoted as A and B, respectively).

At the left end of the beam (fixed):

$$R_A := \frac{-W}{L^3} \cdot (L - a)^2 \cdot (L + 2 \cdot a) \qquad R_A = -500 \cdot lbf$$

$$M_A := \frac{-W \cdot a}{L^2} \cdot (L - a)^2 \qquad M_A = -1.25 \times 10^3 \cdot lbf \cdot ft$$

$$\theta_A := 0 \cdot deg$$

$$y_A := 0 \cdot in$$

At the right end of the beam (fixed):

$$R_B := \frac{-W \cdot a^2}{L^3} \cdot (3 \cdot L - 2 \cdot a) \qquad R_B = -500 \cdot lbf$$

$$M_B := \frac{-W \cdot a^2}{L^2} \cdot (L - a) \qquad M_B = -4.022 \times 10^4 \frac{ft^2 \cdot lb}{s^2}$$

$$\theta_B := 0 \cdot deg$$

$$y_B := 0 \cdot in$$

General formulas and graphs for transverse shear, bending moment, slope and deflection as a function of x

Note: To find the maximum and minimum values of a graphed function, simply **click** once on the graph and read the extreme values to the left of the plot.

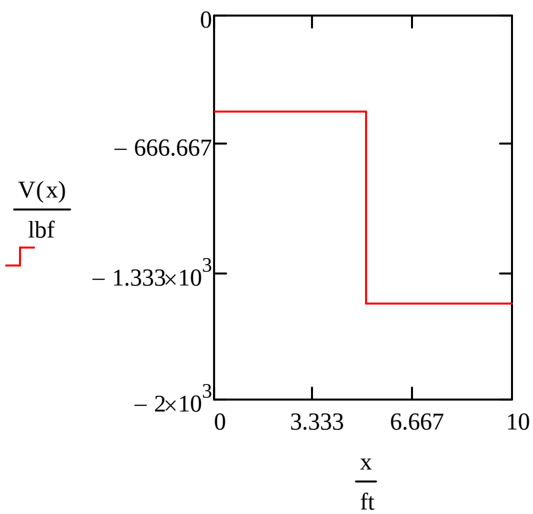
$x := 0 \cdot L, .01 \cdot L .. L$ x ranges from 0 to L, the length of the beam.

$x_1 := 120 \cdot in$ Define a point along the length of the beam.

Transverse shear

$V(x) := R_A - (x > a) \cdot W$

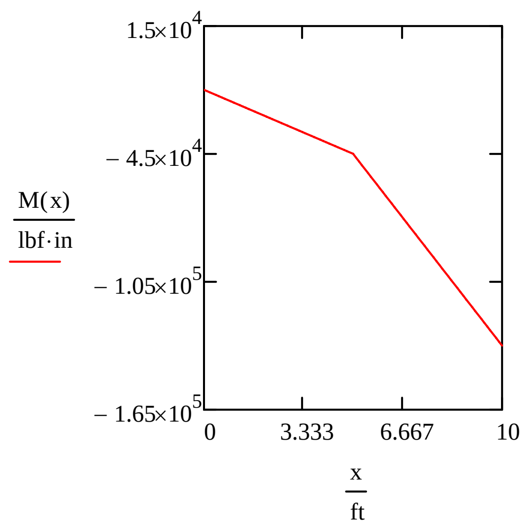
$V(x_1) = -1.5 \times 10^3 \cdot lbf$



Bending moment

$M(x) := M_A + R_A \cdot x - (x > a) \cdot (x - a) \cdot W$

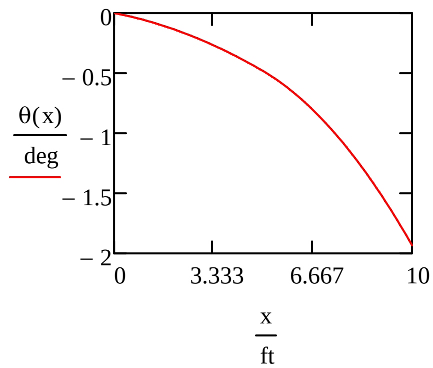
$M(x_1) = -1.125 \times 10^4 \cdot lbf \cdot ft$



Slope

$$\theta(x) := \theta_A + \frac{M_A \cdot x}{E \cdot I} + \frac{R_A \cdot x^2}{2 \cdot E \cdot I} - \frac{(x > a) \cdot (x - a)^2 \cdot W}{2 \cdot E \cdot I}$$

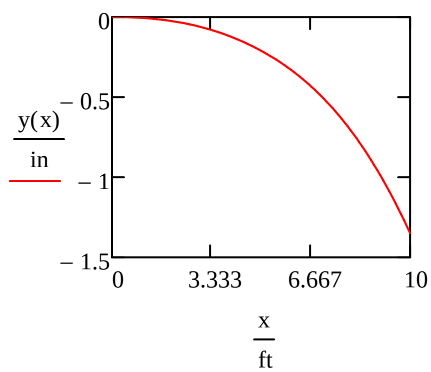
$$\theta(x_1) = -1.933 \cdot deg$$



Deflection

$$y(x) := y_A + \theta_A \cdot x + \frac{M_A \cdot x^2}{2 \cdot E \cdot I} + \frac{R_A \cdot x^3}{6 \cdot E \cdot I} - (x > a) \cdot \left[\frac{W}{6 \cdot E \cdot I} \cdot (x - a)^3 \right]$$

$$y(x_1) = -1.349 \cdot in$$



Selected maximum values of moments and deformations

Note: The signs in this section correspond to direction.

The subscripts **maxpos/neg** refer to the maximum positive or negative value for the given parameters.

At x = a,

$$M_{maxpos} := \frac{2 \cdot W \cdot a^2}{L^3} \cdot (L - a)^2 \qquad M_{maxpos} = 1.25 \times 10^3 \cdot lbf \cdot ft$$

If a < L/2,

$$M_{maxneg} := M_A \cdot \left(a < \frac{L}{2}\right) + M_B \cdot \left(a \geq \frac{L}{2}\right)$$

$$M_{maxneg} = -1.5 \times 10^4 \cdot lbf \cdot in$$

At x equals $\frac{2 \cdot a \cdot L}{L + 2 \cdot a} \cdot \left(a \geq \frac{L}{2}\right) + \frac{2 \cdot (L - a) \cdot L}{L + 2 \cdot (L - a)} \cdot \left(a < \frac{L}{2}\right) = 5 \cdot ft$

$$y_{maxneg} := \frac{-2 \cdot W \cdot (L - a)^2 \cdot a^3}{3 \cdot E \cdot I \cdot (L + 2 \cdot a)^2} \qquad y_{maxneg} = -0.042 \cdot in$$

The subscripts **(p/n)maxval** refer to the maximum magnitude of the most value for specific conditions.

If a = L/2,

$M_{pmaxval} := \frac{W \cdot L}{8}$ $\qquad$ $M_{pmaxval} = 1.5 \times 10^{4} \cdot lbf \cdot in$

If a = L/3,

$M_{nmaxval} := -0.1481 \cdot W \cdot L$ $\qquad$ $M_{nmaxval} = -1.777 \times 10^{4} \cdot lbf \cdot in$

If x = a = L/2,

$y_{nmaxval} := \frac{-W \cdot L^{3}}{192 \cdot E \cdot I}$ $\qquad$ $y_{nmaxval} = -0.042 \cdot in$