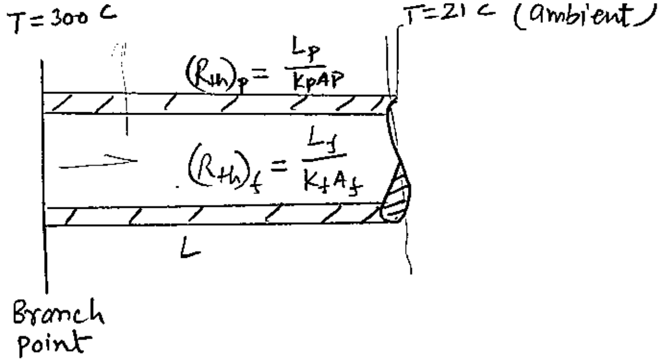
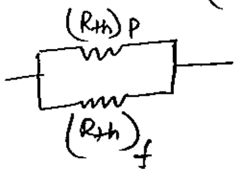


Legends

P – Pipe
f – fluid
R_{th} – Conduction thermal resistance
L – Length
k – thermal conductivity
A – Area
T – Temperature

Equivalent thermal resistance (R_{th})



$$R_{th} = \frac{(R_{th})_p (R_{th})_f}{(R_{th})_p + (R_{th})_f} = \frac{L}{K_p A_p + k_f A_f}$$

$$R_{th} = \frac{L}{\pi D \left(K_p t + \frac{K_f D}{4}\right)} = \frac{4L}{\pi D (4K_p t + K_f D)}$$

for a 10" SCH 40 Pipe ; D = 255 mm, t = 9 mm

$k_f = 0.5$ W/m-k (Hydrocarbon)

$K_p = 50$ W/m.k (Carbon steel)

$R_{th} = \frac{L}{0.35}$

Let us assume temperature will Saturate to ambient after a length L.

Steady state heat flow

$$Q = \frac{\text{Temperature difference}}{\text{Thermal resistance}} = \frac{(300-21)}{\left(\frac{L}{0.35}\right)}$$

$$Q = \frac{97}{L} \simeq \frac{100}{L} \; [W]$$

To satisfy our assumption you require 100 W of steady state heat flow for a piece of 1m pipe filled with Hydrocarbon.

Heat flux at branch point :–

$$q'' = \frac{Q}{A} = \frac{100}{\frac{\pi}{4}(0.273)^2} = \frac{100}{0.06} = 1709 \left[\frac{W}{m^2}\right]$$

$$\text{Biot number (axial)} = \frac{h \times L}{k} = \frac{15 \times 1}{50} = 0.3$$

$$\text{Biot number (transverse)} = \frac{h \times (D/2)}{k} = \frac{15 \times \frac{0.255}{2}}{50} = 0.038$$

steady state temperature based on Lumped capacity analysis

$$t_{\text{steady state}} = t_\infty + \frac{q''}{h} = 21 + \frac{1709}{15} = 135\,^\circ C$$

Average temperature for this pipe

$$t_{average} = \frac{300+21}{2} = \underline{160.5\,^\circ C}$$

$$\%\,error = \frac{160.5 - 135}{160.5} \times 100$$

$$\simeq 16\,\%$$

The error is due to reason that Biot number (axial) is 0.3. To apply Lumped system analysis Biot number shall be less then 0.1.