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# ESTIMATION OF POINT VIBRATION LOADS FROM FIELD MEASUREMENTS FOR INDUSTRIAL PIPING.

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## ABSTRACT

Estimation of point loads on a span for a vibrating pipe has been done with the help of boundary measurement of slope data. The approach is based on the theory of inverse force identification for hyperbolic systems. Depending on the form of the forcing function, which also satisfies some continuity conditions an exact determination, is possible. Though restrictive in its formulation some applications are still possible. A typical case for harmonic excitation of a pulsating reciprocating compressor has been shown as an illustration.

## INTRODUCTION

Failure of an industrial piping due to vibrations is a serious problem and of major concern in the reliability of plant operations. Piping witnesses various vibratory loads throughout its life cycle. These vibrations if not controlled will lead to fatigue failures at points of high stress intensity or could even damage the supports. All these could lead to plant outage or even have more severe consequences like fire and loss of human lives [1]. Thus it is imperative that the piping system along with supports be designed for the vibration loads.

As a part of the design adequacy check the dynamic analysis has to be carried out for the piping system. But the major difficulty in the dealing with the operational vibration is estimation of the forcing function. If the exciting forces can be quantified precisely, the system response can be determined with great accuracy by the existing analytical methods. But unfortunately this is not readily possible in most cases since the vibrations in an operating pipeline are flow induced. The subject is still not completely understood today. The complexity of flow patterns. vortex shedding and mechanism of force coupling between fluid and piping makes the analytical determination of the forcing function extremely difficult. In such a scenario the data in the form of field vibration measurements in conjunction with some analytical methods can provide a basis for estimating the dynamic loads and stresses [2][3].

Some significant development has been made in the mathematical theory of Inverse problems for distributed parameter systems. Particularly for the hyperbolic systems work on the identification of point sources deserves special mention [4-6]. The fundamental objective is to determine the sources of excitation from boundary or interior measurements of some parameters. The main ingredients are spectral properties of differential operators, controllability results and certain properties of some integral operators. There are basically three main steps in the studies : 1)uniqueness, 2)stability and 3)reconstruction. The first two steps deal with the conditions for uniqueness of the sources and their stability for numerical calculations. Reconstruction deals with the methodology for the determination of the sources.
But the major difficulty lies in the application of the theory to real systems. The theory is developed for a simple system whereas the real systems are extremely complex. We are also not sure whether a generalization can be done. However some applications could be found after some extension of the theory. In our study, which is mainly intended for engineering application we have focused on the reconstruction method. In the sequel we shall reproduce the main results of the basic theory and then the application followed by a numerical example.

## NOMENCLATURE

**L** : Length of the pipe span .

**EI** : Elastic Modulus * Area Moment of Inertia - Modulus of Rigidity .
$\mu$ : mass per unit length of pipe/ **EI** .
**x** : space variable : $0 \le x \le L$ .
**t** : time variable .
**T** : Time span .
**u** : Transverse deflection .
$u_x$ : Space derivative of **u** .
$u_t$ : Time derivative of **u** .
$\omega_n$ : Natural frequency for the $n^{th}$ mode .
$\Phi_n$ : Mass - Normalized Eigen Vector for $n^{th}$ mode .
$A_n$ : Amplitude of the eigen-vector for $n^{th}$ mode .
$D_g$ : Convolution Integral Operator .
$D_g^*$ : Adjoint of Operator $D_g$ .
**f(x,t)** : Forcing function .
$\delta(x-x_k)$ : Dirac Delta function at $x_k$ .
$f_k$ : Weights of the Dirac Delta Function for spatial variation of **f(x,t)** .
**g(t)** : Time varying component of the forcing function .
$\dot{g}(t)$ : Time Derivative of **g(t)** .
**M** : Number of Loading points on the pipe.
$\xi$ , $\eta$ : Dimensionless distance **x/L**
$\acute{\omega}_i$ : $i^{th}$ Harmonic Frequency
$C^1(0,T)$ : Space of functions with continuous derivatives in the interval **(0,T)** .
$L^2(0,T)$ : Space of square integrable functions in the interval **(0,T)** .

## MATHEMATICAL BACKGROUND

The main aim of our study is to determine the magnitude of the forcing function acting as point loads on a simply supported pipe span. The forcing function is represented as follows:

$$f(x,t) = \Big(\sum_{k=1}^{M} f_k\delta(x-x_k)\Big)\, g(t) \qquad (1)$$

The forcing function is assumed to be separable in space and time functions. The space varying function is represented by means of Dirac delta functions at **M** points with weights $f_k$.The time varying function **g(t)** is assumed to be in $C^1(0,T)$ and also $g(0) \neq 0$. The function **g(t)** is assumed to be known a-priori. This depends on the nature of excitation source (refer section **APPLICATION** for details).
The material properties of the pipe are assumed to be homogeneous ,isotropic and linearly elastic. Mass property and the size of the pipe is uniform. Shear deformation and damping have been neglected.

The dynamic equation of motion of a simply supported beam as follows :

$$\mu\, u_{tt} + u_{xxxx} = \sum_{k=1}^{M} f_k\, \delta(x - x_k) g(t) \qquad (2)$$

Boundary Conditions (B.C) :

$$u(0,t) = 0 \qquad u(L,t) = 0 \qquad (3)$$

$$u_{xx}(0,t) = 0 \qquad u_{xx}(L,t) = 0 \qquad (4)$$

Initial Conditions (I.C) :

$$u(x,0) = 0 \qquad u_t(x,0) = 0 \qquad (5)$$

The eigen-frequencies of (1) are given as

$$\omega_n = (1/\sqrt{\mu})\,(n\pi/L)^2 \quad \text{(rad/sec)} \qquad (6)$$

The mass normalized eigen-vectors are given as

$$\Phi_n = A_n \sin(n\pi x/L) \qquad n = 1,2,3,4 \ldots. \qquad (7)$$

$$A_n = \sqrt{(2/(\mu L))} \qquad (8)$$

The solution to (2) by mode superposition principle with the B.Cs and I.Cs (3) – (5)and using Duhamel's integral can be written as

$$u(x,t) = \sum_{n=1}^{\infty} A_n \sin(n\pi x/L)(\,1/\omega_n)\Big(A_n\sum_{k=1}^{M} f_k \sin(n\pi x_k/L)\Big)\Big(\int_0^t g(t-s)\sin(\omega_n s)ds\Big) \qquad (9)$$

Let us consider the integral operator $D_g$ as follows.

$$D_g v = \int_0^t \dot{g}(t-s)\, v(s)\, ds \quad \text{for any function } v \in L^2(0,T) . \qquad (10)$$

The adjoint of $D_g$ in $L^2(0,T)$ norm is given by

$$D_g^* v = \int_t^T \dot{g}(s-t)\, v(s)\, ds \qquad (11)$$

Let us now define function $\Psi_m(t)$ as the solution of the integral equation as follows:

$$(g(0) + D_g^*)\Psi_m(t) = \cos(\omega_m t) \qquad (12)$$

where $m = 0,1,2,3, \ldots\ldots.$

The above is a Volterra Equation of the second kind [6][8] and it is well-known that it has a unique solution. We now state a lemma involving the properties of the operator $D_g^*$ which will be used in the proof our key result.

***Lemma 1:*** Let the time duration be defined as $T = k(L^2/\pi)\sqrt{\mu}$ (**k** being a positive integer). Let $\Psi_m(t)$ be a solution of (12).
Then

$$\int_0^T \Big(g(0)\cos(\omega_n t) + \int_0^T \dot{g}(t-s)\cos(\omega_n s)ds\Big)\Psi_m(t) = I_{mn} \qquad (13)$$

where $I_{mn} = 0$ for $m \neq n$ and (14)

$I_{mn} = 0.5T$ for $m=n$ (15)

where $m,n$ are non-negative integers.

In particular $I_{0n} = 0$ for any positive integer $n$ (16)

*Proof :* We have from definition of $D_g^*$

$$\int_0^T ( g(0)\cos(\omega_n t) + \int_0^t \dot{g}(t-s)\cos(\omega_n s)\,ds )\Psi_m(t)dt = \int_0^T (g(0) + D_g^*)\Psi_m(t)\cos(\omega_n t)\,dt$$

$$= \int_0^T \cos(\omega_m t)\cos(\omega_n t)\,dt \quad (17)$$

Thus we have

$I_{mn} = 0.5(\sin((\omega_m + \omega_n)T)/(\omega_m+\omega_n) - \sin((\omega_m - \omega_n)T)/(\omega_m-\omega_n))$ (18)

Let $T = k(L^2/\pi)\sqrt{\mu}$ for any positive integer $k$

Then $I_{mn} = 0$ when $m \neq n$ and $I_{mn} = (0.5T)$ when $m = n$

Thus we prove (14) and (15).

Substituting $m = 0$ in (18) we get (16). ■

With help of the above development we may now formulate our reconstruction strategy in the form of a proposition as follows:

***Proposition 1*** : Let the location of the point sources be known. Then from the boundary measurement of the slope $u_x(0,t)$ or from the displacement measurement of any interior point $u(z_i,t)$ and for a time duration $T = k(L^2/\pi)\sqrt{\mu}$ ($k$ being a positive integer) the load component $f_j$ ($j = 1$ to $M$) can be estimated as follows:

For a single source (i.e $M=1$) :

$$f_1 = \left(\int_0^T u_x(0,t)\,\Psi_0(t)\,dt\right) / \left(\left(\sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\sin(n\pi\xi_1)\right)\left(\int_0^T g(t)\,\Psi_0(t)\,dt\right)\right) \quad (19)$$

Or

$$f_1 = \left(\int_0^T u(z_i,t)\,\Psi_0(t)\,dt\right) / \left(\left(\sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sin(n\pi\xi_1)\right)\left(\int_0^T g(t)\,\Psi_0(t)\,dt\right)\right) \quad (20)$$

For double sources (i.e $M=2$) :

Let

$$C_{11} = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sin(n\pi\xi_1)\ ;\quad C_{12} = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sin(n\pi\xi_2)$$

$$C_{21} = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sin(n\pi\xi_1)\left(\int_0^T g(t)\,\Psi_1(t)\,dt\right) - A_1^2(1/\omega_1^2)\sin(\pi\eta_i)\sin(\pi\xi_1)(0.5T)$$

$$C_{22} = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sin(n\pi\xi_2)\left(\int_0^T g(t)\,\Psi_1(t)\,dt\right) - A_1^2(1/\omega_1^2)\sin(\pi\eta_i)\sin(\pi\xi_2)(0.5T)$$

$$R_1 = \left(\int_0^T u(z_i,t)\,\Psi_0(t)\,dt\right) / \left(\int_0^T g(t)\,\Psi_0(t)\,dt\right)$$

$$R_2 = \int_0^T u(z_i,t)\Psi_1(t)\,dt$$

Then

$$f_1 = (C_{22}/\Delta)R_1 - (C_{12}/\Delta)R_2 \quad (21)$$

$$f_2 = -(C_{21}/\Delta)R_1 + (C_{11}/\Delta)R_2 \quad (22)$$

Where $\Delta = C_{11}C_{22} - C_{12}C_{21}$

Or

Let

$$C_{11} = \sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\sin(n\pi\xi_1)\ ;\quad C_{12} = \sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\sin(n\pi\xi_2)$$

$$C_{21} = \sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\sin(n\pi\xi_1)\left(\int_0^T g(t)\,\Psi_1(t)\,dt\right) - A_1^2(\pi/L)(1/\omega_1^2)\sin(\pi\xi_1)(0.5T)$$

$$C_{22} = \sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\sin(n\pi\xi_2)\left(\int_0^T g(t)\,\Psi_1(t)\,dt\right) - A_1^2(\pi/L)(1/\omega_1^2)\sin(\pi\xi_2)(0.5T)$$

$$R_1 = \left(\int_0^T u_x(0,t)\,\Psi_0(t)\,dt\right) / \left(\int_0^T g(t)\,\Psi_0(t)\,dt\right);\qquad R_2 = \int_0^T u_x(0,t)\,\Psi_1(t)\,dt$$

Then

$$f_1 = (C_{22}/\Delta)R_1 - (C_{12}/\Delta)R_2 \quad (23)$$

$$f_2 = -(C_{21}/\Delta)R_1 + (C_{11}/\Delta)R_2 \quad (24)$$

Where $\Delta = C_{11}C_{22} - C_{21}C_{12}$

***Proof*** : From (9) using integration by parts we obtain the following:

$$u(x,t) = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi x/L)(1/\omega_n^2)\sum_{k=1}^{M} f_k \sin(n\pi\xi_k)\,g(t) - \sum_{n=1}^{\infty} A_n^2 \sin(n\pi x/L)(1/\omega_n^2)\sum_{k=1}^{M} f_k \sin(n\pi\xi_k)( g(0)\cos(\omega_n t) + \int_0^t \dot{g}(t-s)$$

$\cos(\omega n s)\, ds)$ (25)

Also

$$u_x(x,t) = \sum_{n=1}^{\infty} A_n^2 (n\pi/L)\cos(n\pi x/L)(1/\omega_n^2)\sum_{k=1}^{M} f_k \sin(n\pi\xi_k)\, g(t) -$$

$$\sum_{n=1}^{\infty} A_n^2 (n\pi/L)\cos(n\pi x/L)(1/\omega_n^2)\sum_{k=1}^{M} f_k \sin(n\pi\xi_k)( g(0)\cos(\omega_n t) + \int_0^t \dot{g}(t-s)\cos(\omega_n s)ds) \quad (26)$$

For **M =1** we have

$$u_x(0,t) = \sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\, f_1 \sin(n\pi\xi_1)\, g(t) -$$

$$\sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\, f_1 \sin(n\pi\xi_1)( g(0)\cos(\omega_n t) + \int_0^t \dot{g}(t-s) \cos(\omega_n s)\, ds) \quad (27)$$

Multiplying (27) by $\Psi_0(t)$ and integrating we have

$$\int_0^T u_x(0,t)\, \Psi_0(t)\, dt = \sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\, f_1 \sin(n\pi\xi_1) \int_0^T g(t)\, \Psi_0(t)\, dt -$$

$$\sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\, f_1 \sin(n\pi\xi_1)( \int_0^T ( g(0)\cos(\omega_n t) + \int_0^t \dot{g}(t-s) \cos(\omega_n s)\, ds)\Psi_0(t)\, dt \quad (28)$$

Using **Lemma 1** we have from (28)

$$\int_0^T u_x(0,t)\, \Psi_0(t)\, dt = \sum_{n=1}^{\infty} A_n^2 (n\pi /L)(1/\omega_n^2)\, f_1 \sin(n\pi\xi_1) \int_0^T g(t)\, \Psi_0(t)\, dt \quad (29)$$

Thus

$$f_1 = ( \int_0^T u_x(0,t)\, \Psi_0(t)\, dt) / (( \sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\sin(n\pi\xi_1))( \int_0^T g(t)\, \Psi_0(t)\, dt)) \quad (30)$$

and (19) is proved.

For **M =1** we have

$$\int_0^T u(z_i,t)\, \Psi_m(t)\, dt = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2) f_1 \sin(n\pi\xi_1)\, ( \int_0^T g(t)\, \Psi_m(t)\, dt) -$$

$$\sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2) f_1 \sin(n\pi\xi_1)( \int_0^T( g(0)\cos(\omega_n t) + \int_0^t \dot{g}(t-s) \cos(\omega_n s)\, ds)\, \Psi_m(t)dt) \quad (31)$$

Using **Lemma 1** and **m= 0** we have from (31)

$$\int_0^T u(z_i,t)\, \Psi_0(t)\, dt = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2) f_1 \sin(n\pi\xi_1)\, ( \int_0^T g(t)\, \Psi_0(t)\, dt) \quad (32)$$

Hence

$$f_1 = ( \int_0^T u(z_i,t)\, \Psi_0(t)\, dt) / (( \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sin(n\pi\xi_1))( \int_0^T g(t)\, \Psi_0(t)\, dt)) \quad (33)$$

and (20) is proved.

For **M = 2** we have from (25) we obtain

$$u(x,t) = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi x/L)(1/\omega_n^2)\sum_{k=1}^{2} f_k \sin(n\pi\xi_k)\, g(t) -$$

$$\sum_{n=1}^{\infty} A_n^2 \sin(n\pi x/L)(1/\omega_n^2)\sum_{k=1}^{2} f_k \sin(n\pi\xi_k)( g(0)\cos(\omega_n t) + \int_0^t \dot{g}(t-s) \cos(\omega_n s)\, ds) \quad (34)$$

Taking product with $\Psi_m(t)$ and integrating we get

$$\int_0^T u(z_i,t)\, \Psi_m(t)\, dt = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sum_{k=1}^{2} f_k \sin(n\pi\xi_k)( \int_0^T g(t)\, \Psi_m(t)\, dt) -$$

$$\sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sum_{k=1}^{2} f_k \sin(n\pi\xi_k)( \int_0^T( g(0)\cos(\omega_n t) + \int_0^t \dot{g}(t-s) \cos(\omega_n s)\, ds)\, \Psi_m(t)\, dt) \quad (35)$$

For **m = 0** and **m =1** along with **Lemma 1** we obtain

$$\int_0^T u(z_i,t)\, \Psi_0(t)\, dt = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sum_{k=1}^{2} f_k \sin(n\pi\xi_k)( \int_0^T g(t)\, \Psi_0(t)\, dt) \quad (36)$$

$$\int_0^T u(z_i,t)\, \Psi_1(t)\, dt = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sum_{k=1}^{2} f_k \sin(n\pi\xi_k)( \int_0^T g(t)\Psi_1(t)dt) -$$

$$A_1^2 \sin(\pi\eta_i)(1/\omega_1^2)\sum_{k=1}^{2} f_k \sin(\pi\xi_k)( \int_0^T( g(0)\cos(\omega_1 t) + \int_0^t \dot{g}(t-s) \cos(\omega_1 s)\, ds)\, \Psi_1(t)\, dt) \quad (37)$$

From (36) and (37) along with **Lemma 1** and by separating coefficients of $f_1$ and $f_2$ we obtain the following system of linear equations :

$$C_{11} f_1 + C_{12} f_2 = R_1 \qquad (38)$$
$$C_{21} f_1 + C_{22} f_2 = R_2 \qquad (39)$$

Where

$$C_{11} = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sin(n\pi\xi_1) \;;\quad C_{12} = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sin(n\pi\xi_2)$$

$$C_{21} = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sin(n\pi\xi_1)\left(\int_0^T g(t)\,\Psi_1(t)\,dt\right) - A_1^2(1/\omega_1^2)\sin(\pi\eta_i)\sin(\pi\xi_1)(0.5T)$$

$$C_{22} = \sum_{n=1}^{\infty} A_n^2 \sin(n\pi\eta_i)(1/\omega_n^2)\sin(n\pi\xi_2)\left(\int_0^T g(t)\,\Psi_1(t)\,dt\right) - A_1^2(1/\omega_1^2)\sin(\pi\eta_i)\sin(\pi\xi_2)(0.5T)$$

$$R_1 = \left(\int_0^T u(z_i,t)\,\Psi_0(t)\,dt\right) \Big/ \left(\int_0^T g(t)\,\Psi_0(t)\,dt\right)$$

$$R_2 = \int_0^T u(z_i,t)\,\Psi_1(t)\,dt$$

The solution $f_1$ and $f_2$ is as follows :

$$f_1 = (C_{22}/\Delta)R_1 - (C_{12}/\Delta)R_2 \qquad (40)$$

$$f_2 = -(C_{21}/\Delta)R_1 + (C_{11}/\Delta)R_2 \qquad (41)$$

Where $\Delta = C_{11}C_{22} - C_{21}C_{12}$

This proves (21) and (22).

We have for $M = 2$

$$\int_0^T u_x(0,t)\,\Psi_m(t)\,dt = \sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\sum_{k=1}^{2} f_k \sin(n\pi\xi_k)\int_0^T g(t)\,\Psi_m(t)\,dt \;-$$
$$\sum_{n=1}^{\infty} A_n^2 (n\pi x/L)(1/\omega_n^2)\sum_{k=1}^{2} f_k \sin(n\pi\xi_k)\left(\int_0^T \left( g(0)\cos(\omega_n t) + \int_0^t \dot{g}(t-s)\cos(\omega_n s)\,ds\right)\Psi_m(t)\,dt\right) \qquad (42)$$

For **m=0** and **m =1** along with **Lemma 1** and separating $f_1$ and $f_2$ we obtain (23) and (24). ■

## APPLICATION

The above theory is valid for an idealized case and some approximations are necessary for its application to real systems. The configuration of a simply supported pipe header with branches is appropriate for this case. The branch connections act as the point sources of fluid forces. The dominant modes for the header are to be considered. The other condition which bring the restriction on the use of this method is the determination of function **g(t)** (profile of the forcing function). This will depend on the type of excitation. For example it may be taken as the pressure profile for valve closing or opening and it has to be obtained experimentally. But practically it is extremely difficult to determine **g(t)** for general fluid excitations. However for a few cases **g(t)** can be estimated as a series of co-sinusoidal excitations(e.g. pulsations generated from a reciprocating machinery) where the frequencies or harmonics are known (43). In this case we can determine the point forces on the basis of the above mentioned theory as explained below.

We consider a single point source (**M=1**) and two Fourier terms in the time profile. The general case with **p** terms can also be developed on this basis. However for simplification we limit ourselves to **p = 2**.

$$\text{i.e. } g(t) = \sum_{i=1}^{p} Q_i \cos(\acute{\omega}_i t) \qquad (43)$$

***Proposition 2:***

Let **M=1** (single point source) and the time profiles expressed as (43). Let $f_{1i}$ be the force component for the profile component $\cos(\acute{\omega}_i t)$. Let $u_x(0,t)$ be the measured slope time history at the origin.

$$f(x,t) = \sum_{i=1}^{2} f_{1i}\,\delta(x - x_1)\cos(\acute{\omega}_i t) \qquad (44)$$

Where $f_{1i}$ may be obtained by solving the following system of equations.

$$\int_0^T u_x(0,t)\,\Psi_{01}(t)\,dt = \sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\, f_{11}\sin(n\pi\xi_1)\int_0^T \cos(\acute{\omega}_1 t)\Psi_{01}(t)\,dt \;+$$
$$\sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n)A_n\, f_{12}\sin(n\pi\xi_1)\int_0^T\left(\int_0^t \cos(\acute{\omega}_2 (t-s))\sin(\omega_n s)\,ds\right)\Psi_{01}(t)dt \qquad (45)$$

$$\int_0^T u_x(0,t)\,\Psi_{02}(t)\,dt = \sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n^2)\, f_{12}\sin(n\pi\xi_1)\int_0^T \cos(\acute{\omega}_2 t)\Psi_{02}(t)\,dt \;+$$
$$\sum_{n=1}^{\infty} A_n^2 (n\pi/L)(1/\omega_n)A_n\, f_{11}\sin(n\pi\xi_1)\int_0^T\left(\int_0^t \cos(\acute{\omega}_1 (t-s))\sin(\omega_n s)\,ds\right)\Psi_{02}(t)dt \qquad (46)$$

Here $\Psi_{0i}(t)$ is the solution of the Volterra Integral Equation for profile component $\cos(\acute{\omega}_i t)$ .

***Proof:***

This is obtained by the expansion of the $u_x(0,t)$ and integrating with $\Psi_{0i}(t)$ and invoking **Lemma 1** . ■

It is interesting to note that the second term of (45) or (46) is the integral of the product of the slope(for unit point load) and $\Psi_{0i}(t)$.

We now consider the vibrations of the discharge piping header for a reciprocating compressor[Fig.1]. The pipe material is Carbon Steel **A106 GrB** with standard schedule. The branch connection is through a **14" X 8"** weldolet.

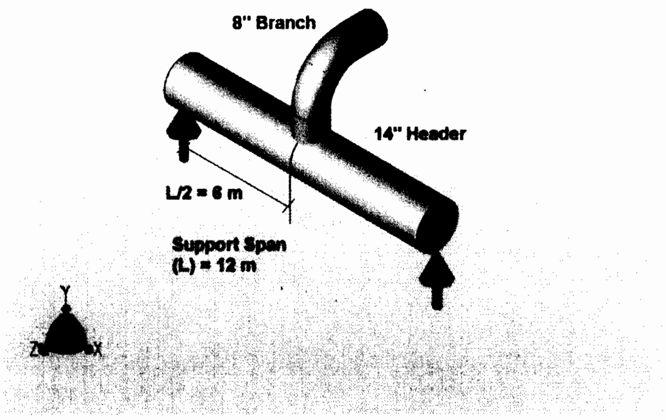


Fig. 1.Compressor Discharge Header Piping

The compressor generates pulsating flow due to the reciprocating nature of movement of the piston. The pulsations originated propagate as pressure waves in the fluid. These waves interact with components like bends , tees etc. and impart dynamic forces on the piping.

The pressure pulsations have a time varying profile as shown in Fig.2. They are almost periodic in nature. The first plot of Fig.2 shows the pressure ratios w.r.t. average pressure vs. time. The frequency domain transform or the time profile is also shown in the second plot of Fig.2.The pressure pulsation peaks occur at the harmonics as shown. The first two harmonics (**4.5 Hz** and **9.0Hz**) have significant contribution and are considered in our analysis . The pressure amplitudes are **10 bar** and **5 bar** respectively for the two harmonics

The pressure profile may be obtained through a pressure probe inserted in the fluid stream. The modern vibration instruments have inbuilt real time data analyzers like F.F.T. The spectral plots may be directly obtained from the instrument or else may be separately analyzed on a separate computer. For a reciprocating machine pressure profile may be calculated using the transient Bernoulli's equation [9].

The analysis begins with the determination of the natural frequencies and the mode shapes of the system. Ten (**10**) modes have been considered. In these modes the effect of the branch is minimal. Which means that the modes are dominantly those of a simply supported beam. The dynamic force is obtained from the pressure profile. The magnitude being dynamic pressure times internal cross-sectional area of the **8"** branch. The point of application of the force is at the weldolet branch junction.

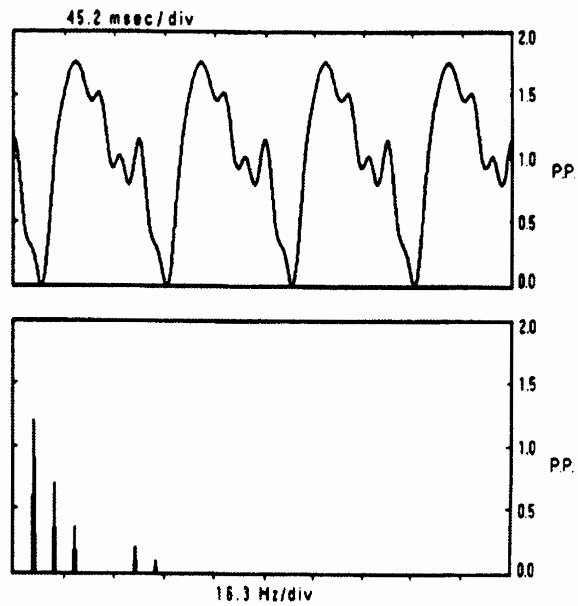


Fig. 2: Pressure Pulsation Pattern

We consider two cases in our analysis. The first case corresponds to the first harmonic excitation (force $\mathbf{f_{\omega 1}}$) and second one consists of both the harmonics (forces $\mathbf{f_{\omega 1}}$ and $\mathbf{f_{\omega 2}}$). The measured values of displacements and slopes are simulated from the time history analysis of (2) with the imposed forces. For both cases **T** is taken as **2200 m secs** .
The analysis has been done for both slope as well as displacement measurement for the second case. The displacement at mid-span has been considered.
The results are shown in Table 1.

| | Measured Parameter | Load No. | Calculated Load(**kN**) | Theoretical. Load(**kN**) |
|---|---|---|---|---|
| | | | | |
| Case 1 | Slope | $\mathbf{f_{\omega 1}}$ | **10.0** | **10.0** |
| | | | | |
| | | | | |
| Case 2 | i. Slope | $\mathbf{f_{\omega 1}}$ | **5.0** | **4.5** |
| | | $\mathbf{f_{\omega 2}}$ | **10.5** | **10.0** |
| | ii. Displ. | $\mathbf{f_{\omega 1}}$ | **5.0** | **4.5** |
| | | $\mathbf{f_{\omega 2}}$ | **10.5** | **10.0** |

Table 1: Comparison of Theoretical and Computed Load values

It is seen that high accuracy in the results are obtained even with a small number of modes. Further improvement using greater number of modes is not worthwhile in view of the rounding errors. Using the computed forces the piping can be analyzed for maximum stress and the maximum support loads. In our case (Case 2) the maximum support reaction is found to be **25 kN** and the maximum stress is found to be about **90%** of the allowable fatigue stress for $\mathbf{10^7}$ cycles. Considering **2.8** Stress intensification factor (SIF) for the **8"** branch with weldolet this even exceeds the allowable stress. The support span needs to be reduced to **8m** for a safe design.

## DISCUSSION AND CONCLUSION

A reconstruction strategy has been shown and can be applied to real systems in a restricted manner. The other aspects like stability and uniqueness can be easily derived from those of the original problem [4-7] and have not been included in our main study. But nevertheless they are absolutely essential for the workability of the method. From the uniqueness theorem we conclude that any set of point loads will result in a unique response. Hence if the response is known with the help of measurements the loads can be uniquely determined.

By stability we have a bound on the variation of the solution subject to the errors in measurement. It means that the error in the solution(values of the load coefficients) will be small if the measurement of the response is done with good accuracy.

The $\mathbf{u_x}$ or slope data can be obtained from displacement measurements at two points close to each other and using the discrete approximation of the derivative. In this case instrument with real time dual channel analyzer has to be used in order to obtain the readings of the two points simultaneously.

In a nutshell it may be said the determination of forces for a vibrating piping system is a difficult task and still an open problem. The method of finding a general solution is stymied due to large number of unknown parameters. Inverse theory is possibly the only method, which can be used, in a somewhat general sense. In our study we have delineated a specialized application of this method.

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