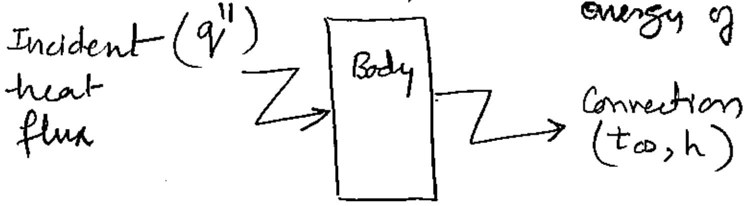
Assuming Lumped Capacity analysis [Internal Conduction resistance negligible compare to outside Convective resistance]

Mathematical modelling of the problem

Writing energy balance gives :-
(Part of the incident heat will lost by convection & remaining part will increase internal energy of body)



$$q''A - hA(t - t_\infty) = mc_p \frac{dt}{d\tau}$$

$$\underbrace{\frac{q''A}{mc_p}}_{\text{assume } a} - \underbrace{\frac{hA}{mc_p}}_{b}\underbrace{(t - t_\infty)}_{\theta} = \frac{dt}{d\tau}$$

$t \rightarrow$ Temperature
$\tau \rightarrow$ Time
$m \rightarrow$ mass
$c_p \rightarrow$ specific heat
$h \rightarrow$ Convective heat transfer coefficient
q'' – flux (heat)
A – surface area.
t_∞ – ambient temp^r.

$$a - b\theta = \frac{dt}{d\tau}$$

$$\theta = (t - t_\infty)$$

$$\frac{d\theta}{d\tau} = \frac{dt}{d\tau}$$

$$a - b\theta = \frac{d\theta}{d\tau}$$

$$\frac{d\theta}{a - b\theta} = d\tau$$

use $\left[\begin{array}{l}\theta_1 = t_\infty - t_\infty = 0 \\ \theta_2 = t - t_\infty\end{array}\right.$

Integrating between initial condition "1" and final condition "2"

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{a - b\theta} = \int d\tau$$

$$\left[\log \frac{a - b\theta_2}{a - b\theta_1}\right] \times \left(-\frac{1}{b}\right) = \tau$$

$$\frac{a - b\theta_2}{\underbrace{a - b\theta_1}_{0}} = e^{-b\tau}$$

$$\frac{a - b\theta_2}{a} = e^{-b\tau}$$

$$1 - \frac{b}{a}\theta_2 = e^{-b\tau}$$

$$\theta_2 = \left(1 - e^{-b\tau}\right)\frac{a}{b}$$

$$t - t_\infty = \frac{a}{b}\left(1 - e^{-b\tau}\right)$$

$$t = t_\infty + \frac{a}{b}\left(1 - \underbrace{e^{-b\tau}}_{0 \text{ when } \tau \to \infty}\right)$$

$$a = \frac{q''A}{mc_p}, \quad b = \frac{hA}{mc_p}$$

$$\frac{a}{b} = \frac{q''_{\text{incident}}}{h}$$

* Now assume a relevant heat flux and heat transfer coefficient to find the value of temperature from equation derived. This equation will give you the temperature after steady state has been reached

Steady state temperature of body ($\tau \to \infty$)

$$t_{\text{steady state}} = t_\infty + \frac{a}{b}$$

$$\boxed{t_{\text{steady state}} = t_\infty + \frac{q''}{h}}$$

\+ This derivation shows how to mathematically model simple case and solve by making simple energy balance.