Geometry (input)

$OD := 6.625 \cdot in$ $\quad wall := 0.28 \cdot in$ $\quad ca := .0625 \cdot in$ $\quad SIFi := 2.218$ $\quad SIFo := 2.218$ $\quad FAC := 1$
(setup parameter)

Geometry Calcs.

$wt := wall - ca \qquad ID := OD - 2 \cdot wt \qquad A := \frac{\pi}{4} \cdot \left(OD^2 - ID^2\right) \qquad I := \frac{\pi}{64} \cdot \left(OD^4 - ID^4\right) \qquad Ro := \frac{OD}{2} \qquad Z := \frac{I}{Ro}$

Loads (input)

$P := 150 \cdot psi \quad Fax := -279 \cdot lbf \quad Mi := 2057 \cdot ft \cdot lbf \quad Mo := -147.9 \cdot ft \cdot lbf \quad T := 202.1 \cdot ft \cdot lbf$

Stresses:

$Slp := \frac{P \cdot ID^2}{OD^2 - ID^2} \qquad Slp = 1031\,psi$

$\sigma hoop := \frac{P \cdot ID}{2 \cdot wt} \qquad \sigma hoop = 2134\,psi$

$\sigma axial := Slp + \frac{Fax}{A} \qquad \frac{Fax}{A} = -64\,psi \qquad \sigma axial = 967\,psi$

$Sb := \frac{\sqrt{(ii \cdot Mi)^2 + (io \cdot Mo)^2}}{Z} \qquad Sb = 8083\,psi \qquad \tau := \frac{T}{2 \cdot Z} \qquad \tau = 179\,psi$

$\sigma := Slp + \frac{Fax}{A} + Sb \qquad \sigma = 9050\,psi$

Set stress intensification factors

$ii := if(FAC \cdot SIFi > 1, FAC \cdot SIFi, 1) \quad ii = 2.218$

$io := if(FAC \cdot SIFo > 1, FAC \cdot SIFo, 1) \quad io = 2.218$

Assemble the stress components required to calculate the three dimensional stress intensity at four locations through the pipe wall.

Calculate the stresses along the pipe diameter which is perpendicular to the resultant bending moment.

1: outside surface, moment causes tension
2: inside surface, moment causes tension
3: inside surface, moment causes compression
4: outside surface, moment causes compression

Intermediate calc's: $Ri := \frac{ID}{2} \qquad Ain := \frac{\pi}{4} \cdot ID^2 \qquad Axs := A$

$axial := P \cdot Ain + Fax \qquad bend := \sqrt{(ii \cdot Mi)^2 + (io \cdot Mo)^2}$

longitudinal stress | hoop stress | radial stress (force this term negative) | shear stress

$$\sigma l := \begin{pmatrix} \frac{axial}{Axs} + \frac{bend \cdot Ro}{I} \\ \frac{axial}{Axs} + \frac{bend \cdot Ri}{I} \\ \frac{axial}{Axs} - \frac{bend \cdot Ri}{I} \\ \frac{axial}{Axs} - \frac{bend \cdot Ro}{I} \end{pmatrix} \qquad \sigma h := \begin{bmatrix} P \cdot \frac{Ri^2}{\left(Ro^2 - Ri^2\right)} \cdot \left(\frac{Ro^2}{Ro^2} + 1\right) \\ P \cdot \frac{Ri^2}{\left(Ro^2 - Ri^2\right)} \cdot \left(\frac{Ro^2}{Ri^2} + 1\right) \\ P \cdot \frac{Ri^2}{\left(Ro^2 - Ri^2\right)} \cdot \left(\frac{Ro^2}{Ri^2} + 1\right) \\ P \cdot \frac{Ri^2}{\left(Ro^2 - Ri^2\right)} \cdot \left(\frac{Ro^2}{Ro^2} + 1\right) \end{bmatrix} \qquad \sigma r := \begin{pmatrix} 0 \\ -P \\ -P \\ 0 \end{pmatrix} \qquad \tau := \begin{pmatrix} \frac{T \cdot Ro}{2 \cdot I} \\ \frac{T \cdot Ri}{2 \cdot I} \\ \frac{T \cdot Ri}{2 \cdot I} \\ \frac{T \cdot Ro}{2 \cdot I} \end{pmatrix}$$

Sa and Sb are the principal 2D stresses in the plane normal to the radial direction

$$Sa(p) := \frac{\sigma l_p + \sigma h_p}{2} + \frac{\sqrt{(\sigma l_p - \sigma h_p)^2 + (2\cdot\tau_p)^2}}{2} \qquad Sb(p) := \frac{\sigma l_p + \sigma h_p}{2} - \frac{\sqrt{(\sigma l_p - \sigma h_p)^2 + (2\cdot\tau_p)^2}}{2} \qquad Sc(p) := \sigma r_p$$

Principal stresses at positions 1 to 4

$$Sq := \begin{pmatrix} Sa(1) & Sa(2) & Sa(3) & Sa(4) \\ Sb(1) & Sb(2) & Sb(3) & Sb(4) \\ Sc(1) & Sc(2) & Sc(3) & Sc(4) \end{pmatrix} \qquad Sq = \begin{pmatrix} 9.055\times10^3 & 8.524\times10^3 & 2.215\times10^3 & 2.066\times10^3 \\ 2.057\times10^3 & 2.208\times10^3 & -6.588\times10^3 & -7.119\times10^3 \\ 0 & -150 & -150 & 0 \end{pmatrix} psi$$

sort stresses at each position (first in ascending order, then in reverse order)

$$Q(p) := sort\left(Sq^{\langle p\rangle}\right) \qquad S(p) := reverse(Q(p))$$

$$S(1) = \begin{pmatrix} 9055 \\ 2057 \\ 0 \end{pmatrix} psi \qquad S(2) = \begin{pmatrix} 8524 \\ 2208 \\ -150 \end{pmatrix} psi \qquad S(3) = \begin{pmatrix} 2215 \\ -150 \\ -6588 \end{pmatrix} psi \qquad S(4) = \begin{pmatrix} 2066 \\ 0 \\ -7119 \end{pmatrix} psi$$

maximum principal stress (S1) at position "p"

$S1(p) := S(p)_1$ $\quad S1(1) = 9055\,psi$ $\quad S1(2) = 8524\,psi$ $\quad S1(3) = 2215\,psi$ $\quad S1(4) = 2066\,psi$

$S2(p) := S(p)_2$ $\quad S2(1) = 2057\,psi$ $\quad S2(2) = 2208\,psi$ $\quad S2(3) = -150\,psi$ $\quad S2(4) = 0\,psi$

minimum principal stress (S3) at position "p"

$S3(p) := S(p)_3$ $\quad S3(1) = 0\,psi$ $\quad S3(2) = -150\,psi$ $\quad S3(3) = -6588\,psi$ $\quad S3(4) = -7119\,psi$

SI:: Maximum Shear Stress Intensity at position "p"

$$SI(p) := S1(p) - S3(p) \qquad \_3DMax := \begin{pmatrix} SI(1) \\ SI(2) \\ SI(3) \\ SI(4) \end{pmatrix} \qquad MaxStressIntensity := max(\_3DMax)$$

$$MaxStressIntensity = 9185\,psi$$

In non-fatigue applications, maximum stress intensity is limited by material yield stress. (Actually, maximum shear stress which is half of the stress intensity is limited by one half of yield.)

Another comparision - von Mises or octahedral shear stress (also known as equivalent stress since this stress calculation is equivalent to the energy of distortion calculation) is limited by yield stress times square root of 2 divided by 3 (.47Sy).

SOct::Octahedral Shear Stress (or Equivalent Stress)

$$SOct(p) := \frac{1}{3}\cdot\sqrt{(S1(p) - S2(p))^2 + (S2(p) - S3(p))^2 + (S3(p) - S1(p))^2}$$

$$OctMax := \begin{pmatrix} SOct(1) \\ SOct(2) \\ SOct(3) \\ SOct(4) \end{pmatrix} \qquad MaxOctShear := \max(OctMax) \qquad MaxOctShear = 3934\,psi$$

$$MaxOctShear = 3934\,psi$$

Following illustrates the four positions where stess is calculated: