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Inclusion of a Support Friction Into a Computerized Solution of a Self-Compensating Pipeline

Thermal elongations of a pipeline are compensated in many cases by bending of pipeline branches. If the pipeline lies on a horizontal rough and flat foundation that bending is influenced by friction forces. Analysis of that influence is given in the paper. A nonlinear, fourth order differential equation with variable coefficients governing the phenomenon is derived and solved numerically as a two-point boundary problem. A version of the solution suitable for a pipeline on discrete supports has been developed. It may be used in conjunction with any existing computer program for pipeline stress analysis. The results demonstrate existence of a very significant additional bending moment due to friction. It may exceed several times the one computed for a pipeline on frictionless foundation.

1 Introduction

A PIPELINE that is laid according to a natural pattern providing for a high bending flexibility (Fig. 1, AB), or has a U-shaped section specially inserted into otherwise straight pipe in order to increase its flexibility (Fig. 1, BC), will have the thermal elongations absorbed (self-compensated) by bending. Frequently such a pipeline lies flat on a horizontal, rough surface and is separated from that surface by discrete supports welded to the pipe, but having a freedom to slide over the surface in both longitudinal and lateral directions, thus providing for freedom of bending necessary for self-compensation. Friction forces that accompany the support sliding influence the bending and, hence, the self-compensation phenomenon. One may expect they act as additional constraints limiting the system flexibility thus increasing the thermal stresses. Analysis of that friction and the resulting increase of stresses in the pipe is the subject of this paper.

2 Derivation of the Mathematical Model

The following well-known properties of friction forces are adopted in further analysis:

1 The sense of the friction force vector at a point is opposite to the sense of the relative velocity vector at the same point of contact between two surfaces sliding one with respect to the other.

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Nomenclature

A = pipe cross section (structural)(cm²)
D = pipe outer dia (cm)
d = displacement (cm)
E = Young's modulus (kgf/cm²)
F = friction force acting on unit length of the pipe
G = weight in a system with friction (Fig. 14) and spring
c = spring constant (kgf/cm)
I = pipe cross-section moment of inertia (cm⁴)
l = length of a straight line sector in the pipeline (cm)
M = bending moment (kgf·cm)
M_0 = maximal bending moment obtained from no-friction elementary solution
q = weight of unit length of a pipe (with a medium inside) (kgf/cm)
$T_1, T_2, \Delta T$ = initial, final temperature, increment of the temperature (deg C)
t = pipe wall thickness (cm)
u = axial displacement (cm)
w = lateral displacement (cm)
w_0 = lateral displacement at $x = 0$
u = axial displacement
v = resultant displacement of infinitesimal section of the pipe (cm)
α = thermal elongation coefficient 1/1 deg C
μ = friction coefficient

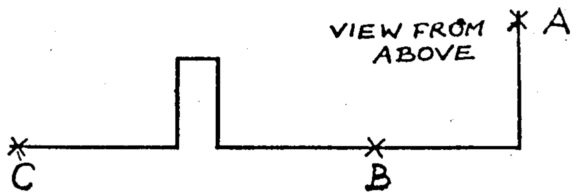

Fig. 1 Typical, self-compensating pipeline

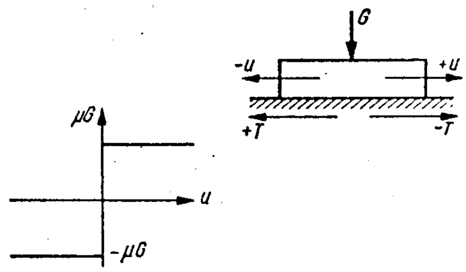

Fig. 2 Dry friction characteristics

2 For dry friction the friction force magnitude is a step function of displacement as shown in Fig. 2. That discontinuity determines the problem as intrinsically nonlinear and eliminates the possibility of using the superposition principle. The final state of deformation reached by the pipeline in the process of heating from temperature T_1 to T_2 will be found by analyzing these deformations and associated forces as they develop, beginning from the initial undeformed state corresponding to temperature T_1.

On the basis of current pipeline design practice the following simplifying assumptions may be made, without loosing the physical sense of the problem:

1 The discrete supports are spaced closely enough to permit their replacement by a continuous support by a flat, horizontal, rigid, and rough foundation having the friction coefficient μ.

2 Unit length of the pipeline weighs q kgf/cm; hence, the friction force acting on that unit length is $F = q\mu$ kgf/cm.

3 $T_2 \leq 200$ deg C, so that one may neglect change to the Young's modulus due to temperature.

4 Tension/compression energy is small compared with bending energy.

2.1 Initial stage. Consider now an infinitesimal pipeline section dx (Fig. 3) at distance x from the fixed support of the pipeline, and let the temperature increase slowly and uniformly in the whole system. The pipeline tends to axially expand by

$$u_T = \alpha \cdot \Delta T \cdot x \tag{1}$$

that generates a counter acting axial friction force cancelling the thermal expansion so that

$$u_F = \frac{F}{EA} \cdot \left(lx - \frac{1}{2} \cdot x^2 \right) \tag{2}$$

No displacement of the dx section will then occur, but since

$$F \leq q\mu \tag{3}$$

that state will last until temperature reaches a threshold increment:

$$\Delta T^* = \frac{q\mu l}{2EA\alpha} \tag{4}$$

After that temperature threshold is exceeded, the pipe begins to move, friction force is fully developed, and its magnitude, but not direction, remains constant. It turns out, conveniently, that ΔT^* is very small in practical cases.

Assume, for instance, the following input values typical for a large diameter pipeline transporting hot water for heating purposes:

$$\begin{aligned} D &= 100 \text{ cm (OD)}, \quad t = 1.6 \text{ cm}, \\ \alpha &= 11.1 \times 10^{-6} \text{ 1/1 deg C (steel)} \\ E &= 2 \times 10^6 \text{ kgf/cm}^2, \\ q &= 11.22 \text{ kgf/cm (pipe full of water)} \\ l &= x_{\max} = 100 \, m = 10^4 \text{ cm} \end{aligned} \tag{5}$$

Substituting it into equation (4), one obtains

$$\Delta T^* = 5.25\mu \tag{6}$$

and for $\mu = 0.4$ (for rusted, not maintained sliding surfaces) $\Delta T^* = 2.12$ deg C. This value is negligible compared with $T_2 \cong 200$ deg C. Hence, one may state with a good accuracy that the pipeline motion due to thermal expansion begins simultaneously with the increase of temperature.

2.2 Stage of fully developed friction. The motion of the pipeline is characterized by: total displacement v, axial displacement u, and lateral displacement w.

According to assumption 4 (section 2.1), equation (1), and the final conclusion of section 2.1, one may deduce that the direction of v tends to become parallel to the pipe axis, as $x \to 0$ ($x = 0$ at the fixed support point), because displacement w vanishes faster (nonlinear function of x) than u (linear function of x), when x decreases. One may, thus, approximate the path of a point of the

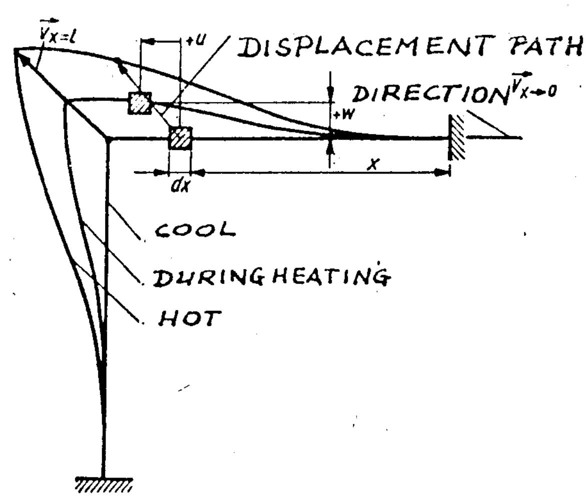

Fig. 3 Self-compensating pipeline in motion due to heating

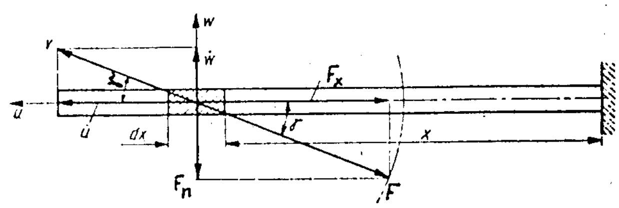

Fig. 4 Displacements, velocities, and forces on an infinitesimal section of the pipeline

pipeline, as a straight line connecting the initial and the final positions of that point.

Hence, for angle γ in Fig. 4

$$\tan\gamma = w/u \tag{7}$$

and the friction force may be resolved into lateral and axial components:

$$F_n = F\cdot\frac{w}{v} = F\cdot w/(u^2+w^2)^{1/2}$$
$$F_x = F\cdot\frac{u}{v} = F\cdot u/(u^2+w^2)^{1/2} \tag{8}$$

Using, again, assumption 4 (section 2.1) to eliminate the influence of F_x on u, one obtains, applying equation (1),

$$F_n = F\cdot w/((\alpha\cdot\Delta T\cdot x)^2 + w^2)^{1/2} \tag{9}$$

The elementary equation of the theory of beam bending may now be written with respect to the friction lateral component treated a continuous loading acting on the pipe in the horizontal plane:

$$EI\cdot\frac{d^4w}{dx^4} = -F_n \tag{10}$$

When equations (8) and (9) and relationship $F = q\mu$ are taken into account, equation (10) becomes:

$$\frac{d^4w}{dx^4} + \frac{q\mu}{\alpha\cdot\Delta T\cdot EI}\cdot\left(x^2+\left(\frac{w}{\alpha\Delta T}\right)^2\right)^{-1/2}\cdot w = 0 \tag{11}$$

It is an ordinary differential equation of the fourth order with a variable coefficient that is a nonlinear function of both dependent and independent variables.

No solution in closed form is known for an equation of this type. Solution has to be sought by means of numerical integration to be carried out specifically for a particular pipeline configuration.

3 Numerical Solution

The equation solution involves four integration constants that are to be determined from the boundary conditions defined at each end of the straight line sector of the self-compensating pipeline. In the theory of differential equations, the determination of these constants from boundary conditions formulated at both ends of the integration interval is known as a so-called two point problem, for which a variety of solution methods, usually iterative, are described in the standard mathematical texts or monographs, such as Fox [1][1] or Hildebrand [2].

The following demonstrates a solution for a particular exemplary pipeline shown in Fig. 5. Due to the symmetry, it is sufficient to consider only one of the branches, for instance branch AB.

Boundary conditions are:

at the corner A:

$$w = \alpha\cdot\Delta T\cdot l = w_0$$
$$\frac{dw}{dx} = 0$$

due to the symmetry.

at point B:

$$w = 0,\qquad \frac{dw}{dx} = 0$$

due to full clamping.

Computations have been carried out for three values of l = 10m, 20m, 100m, typical for design practice and for μ = 0.4. Additionally, μ = 0.1 has been also input for l = 100m, in order to investigate the system sensitivity to the variance of μ.

Results are illustrated in Figs. 6, 7, 8, 9 where the no-friction (μ = 0) solution was also plotted for comparison. The graphs show striking increase of the bending curvature, particularly for the long branch system shown in Fig. 6, for which bending action concentrates clearly in the neighborhood of the corner. The resulting bending moment exceeds many times its value that occurs in no-friction pipeline, as shown in Fig. 7.

Examination of the results plotted in Figs. 6 to 9 suggests the following:

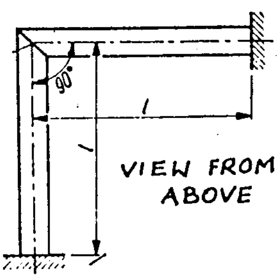

Fig. 5 Exemplary pipeline configuration

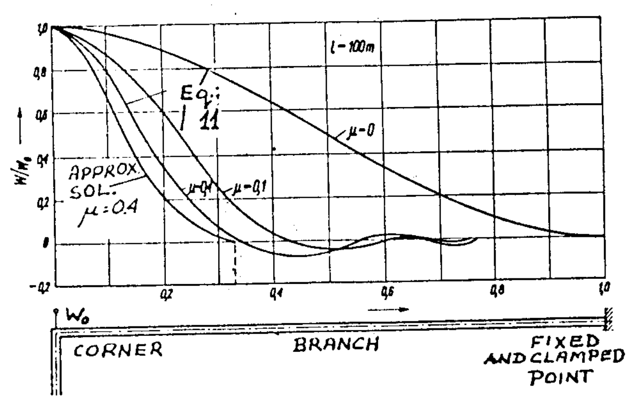

Fig. 6 Elastic line of bending in horizontal plane for different μ

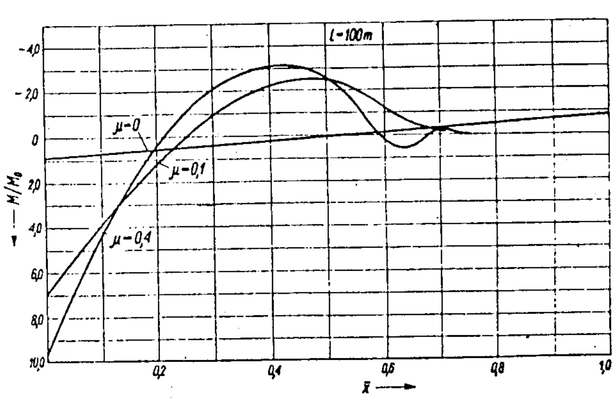

Fig. 7 Ratio of bending moments with friction to maximal bending moment without friction (M_0) versus location along pipe ($\bar{x} = l - x/l$), for l = 100m

[1] Numbers in brackets designate References at end of paper.

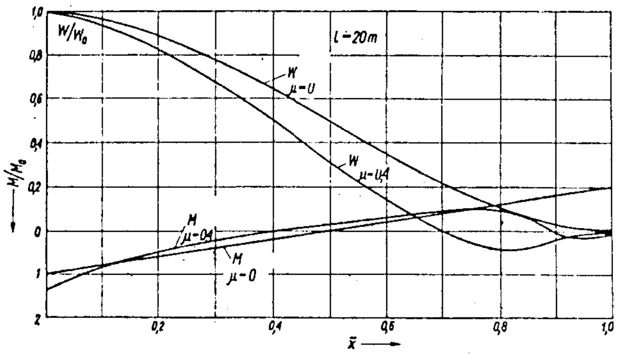

Fig. 8 Elastic line and bending moment for l = 20m

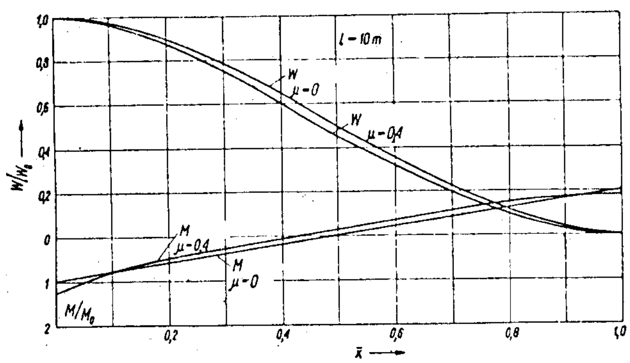

Fig. 9 Elastic line and bending moment for l = 10m

1 Friction may be responsible for a very significant increase of bending stresses in self-compensating heated pipeline.

2 The bending curvature decreases when μ is reduced, but it is a very weak relationship.

4 Approximated, Simplified Method of Solution

Equation (11) has a form similar to that of the classic equation of a beam on an elastic foundation

$$\frac{d^4w}{dx^4} + 4\beta^4 w = 0 \quad (12)$$

For full similarity, one would have to put:

$$4\beta^4 = \frac{q\mu}{\alpha \cdot \Delta T \cdot EI} \cdot \left(x^2 + \left(\frac{w}{\alpha \Delta T}\right)^2\right)^{-1/2} \quad (13)$$

which implies an elastic foundation of the stiffness varying as function of both x and w. It is a weak function, however. Therefore, one may fix β at the value which occurs for $x = l$, at the corner. Thus,

$$\beta = \left(\frac{q\mu}{4}\right)^{1/4} \cdot (\sqrt{2}\, EIl\alpha\Delta T)^{-1/4} \quad (14)$$

and maximal bending moment, known from the solution for the beam on elastic foundation occurs at $x = l$, and is determined by

$$M_{\max} = w_0 \cdot 2\beta^2 EI \quad (15)$$

where

$$w_0 = (w)_{x=l} = \alpha \cdot \Delta T \cdot l$$

It has to be clearly understood that the "elastic foundation" referred to above, is a fictitious foundation created to represent the action of the lateral friction component F_n (section 2.2) acting in horizontal plane. The real foundation on which the pipe rests is, and remains, rigid.

Comparing moment from equation (15) with the one corresponding to an elementary solution for the pipeline with no friction, one obtains ratio η:

$$\eta = (M_{\max})_{\mu \neq 0}/(M_{\max})_{\mu = 0}$$

$$= \frac{1}{6} \cdot (q\mu)^{1/2} \cdot (\sqrt{2}EI\alpha\Delta T)^{-1/2} l^{3/2} \quad (16)$$

Discrepancy between the results of numerical integration of exact equation (11) and the simplified solution represented by equation (16) is plotted in Fig. 10. In the region 40m $\leq l \leq$ 100m, maximal error in terms of ratio η is 18 percent of the more accurate value; mean error is 10 percent. The error gives over estimation of the bending moment.

In view of the above and the uncertainty of the input data regarding friction coefficient μ, one may accept equation (16) as sufficiently accurate for engineering practice. It will, however, apply only to such relatively rare pipeline designs in which the pipe rests directly on the surface or is supported by discrete supports spaced so closely that they may be idealized as continuous support.

The case of discrete supports with friction spaced wide apart, which is more relevant to the actual design practice, is discussed in the following section.

5 Adaptation of the Algorithm to the Discrete Supports and Existing Computerized Procedures.

In section 4, action of the dry friction force has been idealized by a fictitious elastic foundation characterized by a coefficient β determined by equation (13). Such elastic foundation may be discretized to a set of elastic supports (springs). Spring constant

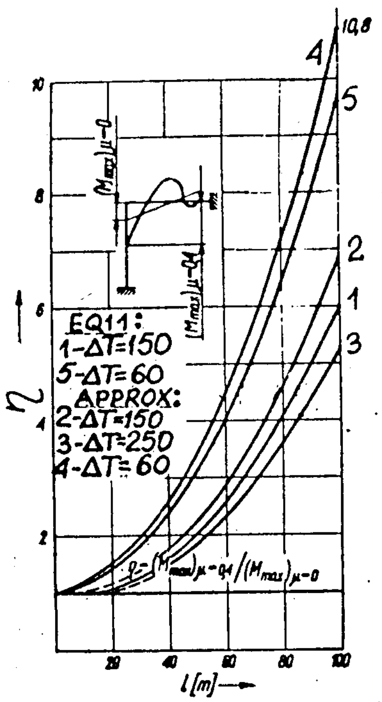

Fig. 10 Comparison of η-ratios obtained from solution of equation (11) and the simplified solution

c of the support will depend on the elastic constant of the elastic foundation k and space h between the supports:

$$c = k \cdot h \tag{17}$$

where k (unit of force/unit of length sq.) is related to β according to theory of a beam on elastic foundation:

$$k = 4\beta^4 \cdot EI \tag{18}$$

Considering equations (8), (10), (12), (17), and (18), one obtains for a support number j:

$$c_j = q \cdot \mu \cdot h \cdot (u_j^2 + w_j^2)^{-1/2} \tag{19}$$

The dimension of c_j is, consequently, (unit of force/unit of length) as a spring constant. For those sectors of pipeline which are attached to a fixed point one may use the approximation:

$$u_j = \alpha \cdot \Delta T \cdot s_j \tag{20}$$

(according to equation (1)) where s_j is measured from the fixed point to the support j (see Fig. 11); thus u_j becomes a constant for support j. Lateral deflection w_j is unknown; so is axial displacement u_j, but only for those sectors which are not attached to a fixed point, for instance, the central sector in self-compensating Z-layout.

One has to emphasize, again, at this point that the above elastic supports (springs) are fictitious ones introduced for sole representation of the lateral, horizontal action of friction on the real sliding supports that are still assumed rigid with respect to vertical forces. An example of a self-compensating pipeline so idealized is shown in Fig. 11.

Pipeline on so-defined elastic supports readily lends itself to a routine computer analysis by means of well-established standard programs. A solution may now be obtained by iteration, in the following steps:

1 Assume w_j, u_j for all supports[2] (assigning to u_j the value given by equation (20) for pipe sectors attached to a fixed point).

2 Determine c_j for each spring j according to equation (19).

3 Use any computer program available that will solve a heated pipeline restrained by discrete springs (example in Fig. 11), each of the springs having a constant c_j, for displacements and stresses.

4 Repeat steps 2, 3, and 4, substituting new w_j, u_j, calculated in step 3, into equation (19) in step 2, until w_j, u_j (or c_j) converge. The steps are illustrated by a flowchart in Fig. 12. The process is rapidly converging due to c_j being a weak function (equation (19)) of u_j, w_j, and for typical layouts sufficient accuracy is obtained in 3 to 5 iterations. This makes the method quite practical even if data transfer to and from a standard computer program used in step 3 (box 3 in Fig. 12) is to be done by hand.

The above is illustrated by a numerical example shown in Figs. 11 and 13. ELAS computer program[3] (reference [3]) has been applied in step 3. It took 4.9 sec of CPU time on a CDC 6600 computer per iteration and converged in 4 iterations to accuracy $(((w_j)_{new} - (w_j)_{old})/w_{j\,new}) \leq 0.015$. Maximum bending moment and stress reached 3.43 times the corresponding values for the frictionless solution.

[2] To avoid singularity avoid $u_j^2 + w_j^2 = 0$.
[3] General purpose program for structural analysis.

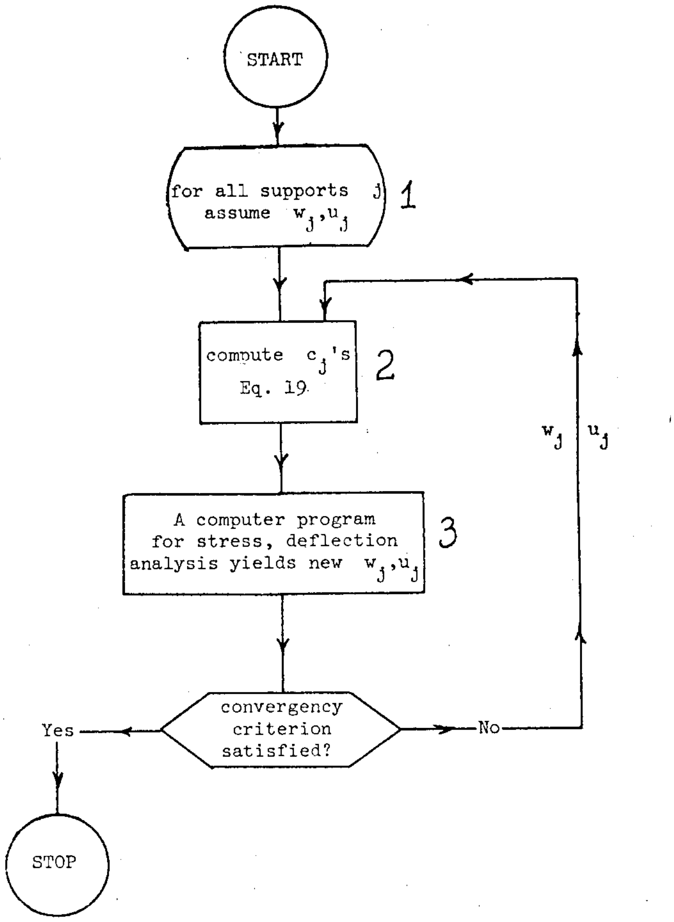

Fig. 12 Flow chart of the iterative computation

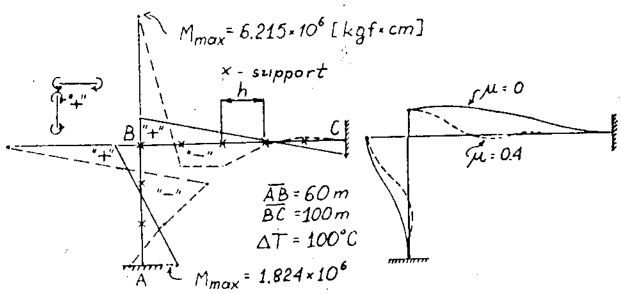

Fig. 13 Example of the deflections and bending moments for a pipeline on supports with friction (dashed line) and without friction (continuous line) (all data as in equation (5), except length; support spacing h = 20 m)

a) Bending moments b) Deflections W

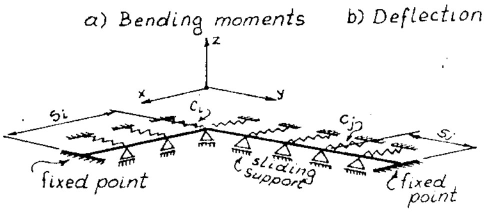

Fig. 11 Perspective view on a pipeline (Z-layout) on discrete supports with friction sliding in horizontal plane, idealized for computer analysis. Springs c_j represent horizontal action of the friction on the sliding supports.

6 Undeterministic Character of the Problem

It is a well-known property of an elastic system with dry friction

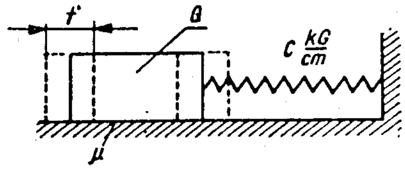

Fig. 14 Elastic system with friction

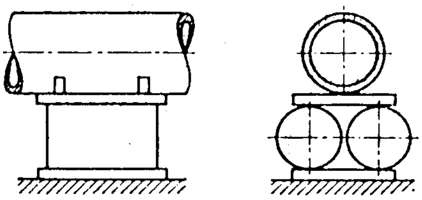

Fig. 15 Pipe on rollers positioned to eliminate F_n

constraints that it may attain several static equilibrium positions within limits determined by the friction forces. For instance, a simple system shown in Fig. 14 will stay in equilibrium for any distance f provided

$$f \cdot C \leq G \cdot \mu$$

Displacement d resulting from a static load P, $d = \frac{P}{C}$, determines merely the center of the interval $2f$. Similar behavior is to be expected in the pipeline problem discussed here.

In particular, referring to Fig. 6, one may regard the elastic line of bending for $\mu = 0.4$ as resulting from an idealized deformation process in which friction forces stay constant at all points during the whole time, while temperature rises slowly and monotonically. In reality, unavoidable vibrations of the foundations will cause random fluctuations of the vertical pressure on the supports and resulting release of the friction constraints. Each release will let the elastic pipeline creep toward the elastic bending line, corresponding to $\mu = 0$, that is, toward lower level of the elastic energy. Similar results will have the temperature fluctuations due to normal or unscheduled pipeline modes of operation.

Accordingly, the true elastic line will be established between the curves for $\mu = 0$, and $\mu = 0.4$, its exact position being a random variable. Similarly, true bending moment will be bounded by the solutions without and with friction

$$(M)_{\mu=0} \leq M_{\text{true}} \leq (M)_{\mu\neq 0}$$

The solution discussed here is, thus, to be understood as an upper bound of the bending moment, the exact value of which is dependent on random factors.

The whole problem then has clearly not a deterministic, but a stochastic character.

Conclusions

Support friction may increase bending moment in self-compensating heated pipeline several times in comparison to an ideal case with no friction.

A practical, approximated analysis with friction that makes use of existing structural analysis computer programs is proposed.

The problem has a stochastic character, analyses with and without friction produce merely the lower and upper bounds on the bending moment value, which is a random variable within these bounds.

One may decrease the bending moment by reducing the friction coefficient μ but it has to be a radical reduction, such as the one resulting from replacing the sliding surfaces by rollers. Interestingly enough rollers should be positioned as shown in Fig. 15 to effectively eliminate lateral friction forces (F_n), not the axial component F_x, which is essentially harmless.

References

1 Fox, L., *The Numerical Solution of Two Point Boundary Problems in Ordinary Differential Equations*, Oxford Clarendon Press, London, 1957.

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3 Utku, S., and Akyuz, F. A., "ELAS—A General Purpose Computer for the Equilibrium Problems of Linear Structures," NASA TR 32-1240, Feb. 1968.