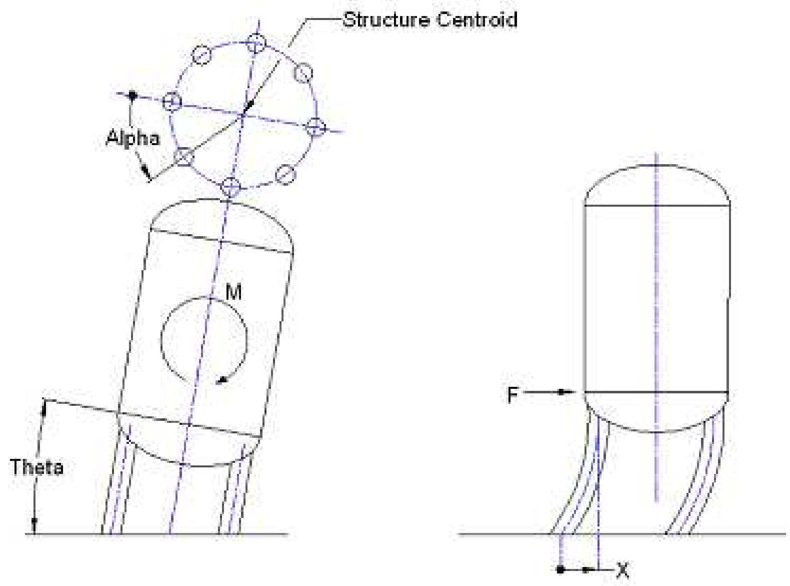
# Analysis of Vertical Vessels Supported on Legs
# By: Ray Delaforce

A common method of supporting short vertical vessels is to employ legs, usually comprising standard structural sections such as rolled steel angles, or 'I' beams. Tall vessels are supported more commonly on cylindrical skirts, but legs are convenient for shorter vertical vessels.

Legs however, have a smaller stiffness, especially with regard to the horizontal movement, or translation of the vessel under the action of horizontal forces from wind and seismic loads. Legs are a little more complex to analyse than skirts for a number of reasons that are enumerated throughout this discussion.

<u>Moment acting on the vessel</u>

Let us consider what happens when a vertical vessel is subject a wind moment and a horizontal wind shear force acting the top of the legs where they are attached to the vessel. This is illustrated in figure 1.




(a) (b)

Figure 1

As can be seen, the moment M rotates the vessel through and angle **θ**. This would be the same situation if we were considering a skirt. However, the complication is what happens as shown if figure 1(b) above. The horizontal force F translates the vessel horizontally. This part is a little more complex to analyse.

Let us first consider what happens as shown figure 1(a) above. In the illustration, the plan view suggests there are 8 legs. However, any number of equally spaced legs can be treated as discussed below.

If the legs each have the same cross-sectional area, and are located about the vessel axis symmetrically around the vessel, the common centroid (of all the legs) is located at the axis of the vessel. The proof is not offered here, but it is trivial in nature.

For any elastic body, the following equation holds true:

Force = Deflection x Stiffness

This is a simple manifestation of Hook's Law.

From figure 1(a) above, it can be seen that the vertical deflection is given by the equation:

Force on one $F = \theta \cdot R \cdot K \cdot \cos(\alpha)$ leg

Where:

| | |
|---|---|
| Θ | Rotation of the vessel from the vertical (radians) |
| α | Angle of the leg around the vessel |
| R | Radius of the circle containing the legs |
| K | Vertical stiffness of the leg (force per unit length) |

We can now find the moment required to promote the tilting of the vessel from the vertical, taking into account the resistance offered by all the legs as follows:

Moment on all legs $M = \theta \cdot R^2 \cdot K \cdot \sum \cos(\alpha)^2$

From this, it is easy to determine the angle of tilt as follows

Angle of tilt $\theta = \dfrac{M}{R^2 \cdot K \cdot \sum \cos(\alpha)^2}$

But, it can be shown that

$$\sum_{i=1}^{n} \cos\left(\frac{2 \cdot \pi}{n} \cdot i\right)^2 = \frac{n}{2}$$

Force on one leg after substituting the above equations becomes (where n is the number of legs):

$$F = \frac{2 \cdot M}{n \cdot R} \cdot \cos(\alpha) \qquad \text{and} \qquad \sigma = \frac{2 \cdot M}{n \cdot A_{leg} \cdot R} \cdot \cos(\alpha)$$

In other software written by our competitors, the force on the leg resulting from the overturning moment is usually given by:

$$F = \frac{2 \cdot M}{n \cdot R}$$

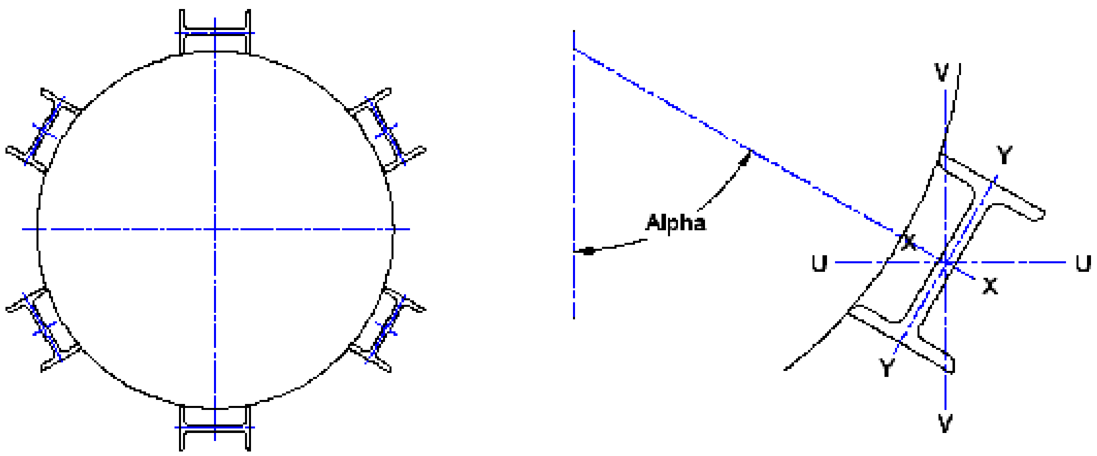
Although this is conservative, the missing Cos(α) term makes it less accurate, it also gives too high a computed stress in the final leg calculation. So, the final compressive (vertical) stress in any given leg is:

$$F = \frac{2 \cdot M}{n \cdot R} \cdot \cos(\alpha)$$

Let us look now at the effects of the wind or seismic horizontal shear force shown in figure 1(b).

<u>Horizontal shear force acting on the vessel</u>

Before we get into the forces acting, let us look at a typical leg arrangement, where 'I' beams are used for the leg components:




(a) b)

Figure 2

Figure 2(a) shows a typical arrangement for these legs. Figure 2(b) shows any leg where its position around the vessel is denoted by the angle α. If we assume the force on the vessel is coming from the south, then the moment of resistance bends the leg about the U-U axis (see figure 2(b).

The horizontal force on any leg depends on the translation ‘X’, and the elastic resistance of the leg K.

$$F_{leg} = X \cdot K_{leg}$$

Where:

$K_{leg}$ is the stiffness of the leg to resist horizontal translation

The total force resisted by all the legs is given by:

$$F = X \cdot \sum K_{leg}$$

Because all the legs experience the same translation (‘X’), the force on the leg is simply give by:

$$F_{leg} = F \cdot \frac{K_{leg}}{\sum K_{leg}}$$

Now, for a conservative approach, we make the following assumptions:

- The legs are pin jointed at the base (bottom of the legs)
- The top of the legs cannot rotate or twist

Thus, the stiffness of any leg is given by:

$$F_{leg} = F \cdot \frac{I_{uu}}{\sum I_{uu}}$$

The value of $I_{uu}$ and $I_{vv}$ can be easily calculated as follows:

$$I_{UU} = I_{YY} \cdot \cos(\alpha)^2 + I_{XX} \cdot \sin(\alpha)^2$$

Finally, the horizontal force on one leg is given by:

$$F_{leg} = F \cdot \frac{I_{YY} \cdot \cos(\alpha)^2 + I_{XX} \cdot \sin(\alpha)^2}{\sum \left(I_{YY} \cdot \cos(\alpha)^2 + I_{XX} \cdot \sin(\alpha)^2\right)}$$

Now, we have a force acting in the line of action of the shear force acting on the vessel. This force is given by the equation shown just above. To analyse the leg in order to perform the unity check in accordance with AISC we have to break the force into its two components about the XX and YY axis. This is shown in figure 3:

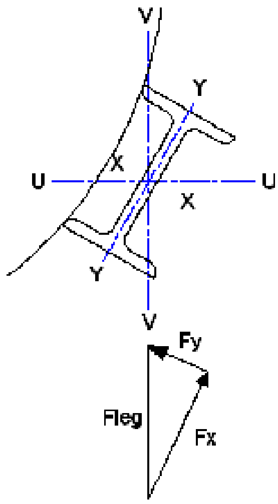



Figure 3

It can easily be shown that:

$$Fy = F_{leg} \cdot \cos(\alpha) \qquad \text{and} \qquad Fx = F_{leg} \cdot \sin(\alpha)$$

Summarising, we can now state the following for any one leg:

Vertical stress from the overturning moment:

$$\sigma v = \frac{4 \cdot M \cdot \cos(\alpha)}{\pi \cdot Aleg}$$

Bending stress, bending the leg about the XX direction is:

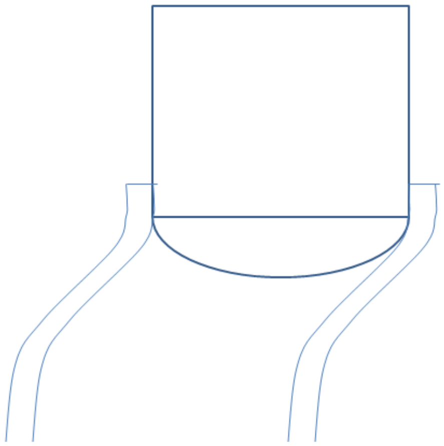
$$\sigma x = \frac{F_{leg} \cdot \sin(\alpha)}{Z_{YY}}$$

Bending stress, bending the leg about the YY direction is:

$$\sigma y = \frac{F_{leg} \cdot \cos(\alpha)}{Z_{XX}}$$

Referring the unity check equation given in *Manual of Steel Construction – Allowable Stress Design 9th edition*, equation H1-1 to H1-3 inclusive, the equations used there are used in PV Elite.

The legs are considered to act as guided cantilevers taking deforming under load in this form:



As can be seen from the diagram, there is moment imposed on the shell from the top of the leg as a result of the deformation of the leg. Added to that is the vertical load from the weight of the vessel, and the overturning moment from the lateral force from the wind and/of seismic force. The stresses in the shell from these forces and moments can be analysed using a procedure such as WRC 107. Thus the analysis can be completed to ensure the legs and shell are adequately designed.