Vibrations of Vertical Pressure Vessels

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This paper is primarily concerned with the vibration of vertical pressure vessels known as columns or towers.

The procedure for estimating the period of first mode of vibration for columns which are the same diameter and thickness for their entire length is outlined. A graph is included for this purpose which recommends limits between vessels considered to be static structures and those considered dynamic.

A method for designing vessels considered as dynamic structures is described as well as a detailed procedure for estimating the period of vibration of multithickness (stepped shell) vessels and/or vessels built to two or more diameters with conical transitions where the difference in diameter is small.

There is a brief resume of the "Karman vortexes" effect and a discussion regarding vibration damping by liquid loading and the benefit of ladders and platforms which help reduce the effect of periodic eddy shedding.

The design procedure outlined will be useful to the practical vessel designer confronted with the task of investigating vibration possibilities in vertical pressure vessels.

Introduction

For many years it was customary to apply guy wires to tall ,slender pressure vessels. In, recent years, refinery and petro-chemical officials have demanded self-supporting vessels from the standpoint of plant appearance and safety.

In order to design a self-supporting vessel of this type, the following problems must be carefully analyzed:

1 When it is necessary to deviate from the common practice of designing a vertical vessel as a static structure and consider it as a dynamic structure?

2 What is the most practical method for designing to meet dynamic conditions?

3 Does the method used produce consistent results and does it provide additional strength to resist the force due to the mass-acceleration resulting from the motion of the vessel ?

4 Is the period of vibration of the dynamically designed vessel such that prevailing winds are not apt to cause excessive movement?

5 Are the external attachments (such as piping, ladders, and platforms) distributed all around the vessel to guard against resonance due to eddy shedding in the "Karman vortex trail" at critical wind velocities?

These problems will be discussed during the outline of a design procedure presented in this paper.

Before proceeding, it should be pointed out that vessel vibrations induced by earthquakes are infrequent in occurrence and this paper is more concerned with vibrations induced by wind or other forces which may occur every day or many times during the day may depending upon the location.

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Nomenclature

f = lowest natural frequency of vibration, cycles per second

$T = \frac{1}{f}$ = period of vibration, sec

g = acceleration due to gravity

W = total weight of vessel or vessel section above horizontal plane under construction, lb

W_S = shear load at end of section, lb

w = unit weight; lb/ft

w' = weight of vessel element or internal part, lb

L = total length, ft

l = length of element or section, ft

D = vessel diameter, ft

d = vessel diameter, in.

t = thickness of vessel shell, in

$h = \frac{t}{12}$ = thickness of vessel shell, ft

y = deflection of element or section, ft

y' = distance from e. g. of vessel element or internal part (of weight w') to seam or horizontal plane under consideration, ft

F_s = seismic factor

E' = 4320 X 10^6 = modulus of elasticity for steel, Ib/ft^2

E = welded joint efficiency

$I = \frac{\pi}{8} D^3 h$ moment of inertia of vessel shell cross-sectional area, ft^4

V = velocity, ft/sec

k = Strouhal number

θ = end slope of element in bending as a cantilever beam, radian[4] (tan θ = θ)

P = internal pressure, psig

S = allowable stress of vessel material, psi

M = moment about vessel seam or horizontal plane under consideration, lb - ft

M_T = moment at end of vessel section resulting from weight of sections to the right section under consideration

C = corrosion allowance

R = Reynolds number

Design

It is customary for most vessel designers to establish the minimum vessel shell and trend thickness according to the pressure temperature conditions and then calculate the thickness required at the bottom head seam due to bending moments imposed by wind or earthquake forces [9].' Stresses in the longitudinal direction are involved nod the following notation may be used to summarize the thickness required:

$$t = \left| \pm \frac{Pd}{4SE + 0.8P} \pm \frac{48M}{\pi d^2 SE} - \frac{W}{\pi dSE} \right| + C \tag{1}$$

The terms within the absolute value signs are positive for tensile stresses and negative for compressive stresses. The first term gives the thickness required for the longitudinal stress resulting from internal pressure and is positive for pressures above atmospheric and negative for pressures below atmospheric. The second term is the thickness required to resist the longitudinal bending stress and both positive and negative values exist at the same time. The third term is the thickness required for the weight of the vessel above the seam being investigated and, since this is a compressive stress, it has a negative value. The combination giving the highest value establishes the thickness required to resist the longitudinal stresses.

Consider equation (1) for a typical vessel operating at an internally pressure greater than atmospheric:

$$t = \left| +0.275 \pm 0.307 - 0.063 \right| + 0.125$$

The required thickness within the absolute value signs will have two values; namely, +0.519 in. and -0.095 in. Therefore the minimum thickness required is 0.519 + 0.125 in. corrosion allowance = 0.644 in.

Next consider equation (1) to appear as follows for the same vessel operating under vacuum conditions:

$$t = \left| -0.123 \pm 0.307 - 0.063 \right| + 0.125$$

For this case, the two values within the absolute value signs are -0.493 and + 0.121 in. resulting in a minimum thickness of 0.493 + 0.125 in. = 0.618 in.

As previously stated, the moment M is the longitudinal bending moment due to wind or earthquake, either of which may be combined with eccentric loads imposed by mounting heavy equipment on the vessel. All designers are accustomed to evaluating moments due to eccentric and wind loads, but there are a few who may not be familiar with the method used for estimating moments due to earthquake. Therefore, the following brief outline is presented because this method is recommended as a design procedure for vessels where dynamic considerations are required. The weight of each vessel element (shell, head, tray, or internal part) is calculated. and then multiplied by the vertical distance from the circumferential seam (or horizontal plane) under consideration to the center of gravity of the element. The summation of the moments so found is multiplied by the seismic factor for the area where the vessel is to operate, thereby yielding a moment due to earthquake or seismic disturbance. For vessels, the seismic factor will usually have a value of 0.03 to 0.12, depending upon the geographical location. Expressed mathematically,

$$M = F_3 \sum w' y' \tag{2}$$

After the vessel has been designed in the regular manner (considered as a static structure) it should be investigated regarding its possible behavior under vibration conditions. If the vessel shell is of constant diameter and thickness for its full length, the period of vibration maybe easily found from the graph shown in Fig. 1. This graph is plotted from the general formula for the period of the first mode of vibration of a cantilever beam [7]:

$$\begin{aligned} T &= \frac{2\pi}{3.52} \left(\frac{wL^4}{E'Ig} \right)^{1/2} \\ &= 1.785 \left(\frac{wL^4}{E'Ig} \right)^{1/2} \end{aligned} \tag{3}$$

For a steel cylindrical shell, equation (3) may be written:

$$T = 7.64 x 10^{-6} \left(\frac{wL^4}{D^3 h} \right)^{1/2} \tag{4}$$

By rewriting equation (4) in the form:

$$T = 7.64 x 10^{-6} \left(\frac{L}{D} \right)^2 \left(\frac{wD}{h} \right)^{1/2} \tag{5}$$

the variables (*L/D*) and (*w*D/*h*) are used as parameters to plot the graph in Fig. 1.

One of the first graphs of equation (4) was issued by a major oil company for their refinery work. In its original form, all vessels having a period of vibration over 0.4 sec were ordered designed as dynamic structures and those having a vibration period of 0.4 sec or less were ordered designed as static structures. Experience has shown that a more practical limit for this division is a line drawn from 0.4 sec at the extreme left of the graph to 0.8 sec at the extreme right and considering vessels having a period of vibration above this line to require dynamic consideration and those below to require designing as a static structure. The reason for revising the former limit is the fact that many vessels having small (*L/D*) ratios and large values of (*wD/h*) have given satisfactory service although their period of vibration exceeded 0.4 sec. In general, vessels having an (*L/D*) ratio less than 15 are not apt to be critical from a vibration, standpoint. One exception to this statement, unofficially reported to the author, involved two vessels operating near a railroad whereby they were vibrated by railroad equipment. Both vessels had a period of vibration considerably less than 0.4 sec and their frequency probably coincided with the frequency of the exciting force, thereby causing resonance. This type of response is difficult, if not impossible, to predict accurately and should be considered as a special case.

If investigation indicates that the vessel should be designed as a dynamic structure, the method of seismic analogy is recommended. This method consists of designing the vessel for earthquake conditions using a seismic factor F_3 = 0.20, regardless of the geographical location. In most cases, the vessel will have thicker shell and head material in the lower section. As an example, consider a vessel 10 ft 0 in. diameter by $^{13}/_{16}$ in. thick by 190 ft 0 in. high which has an (*L/D*) ratio of 19, and period of vibration (after being designed as a static structure) of 1.65 sec. This vessel, when designed as a dynamic structure by the method of seismic analogy, resulted in a shell thickness of $^{13}/_{16}$ in. for the upper 137 ft 0 in. and three 1ower sections consisting of $^{7}/_{8}$, $^{15}/_{16}$, and 1-in. thick material (the supporting skirt increased from 1 to l $^{9}/_{16}$ in.). The period of vibration was reduced to approximately 1.4 sec.

Whereas the application of this method actually consists of trial and error, the experienced pressure vessel designer becomes very proficient in estimating how far down the vessel he can utilize the material thickness which is based on pressure-temperature requirements, as well as the length of successive sections of thicker material. It is usually unnecessary to carry the seismic analogy into the design of the anchor bolts because this method is

[1] Numbers in brackets designate References at end of paper.

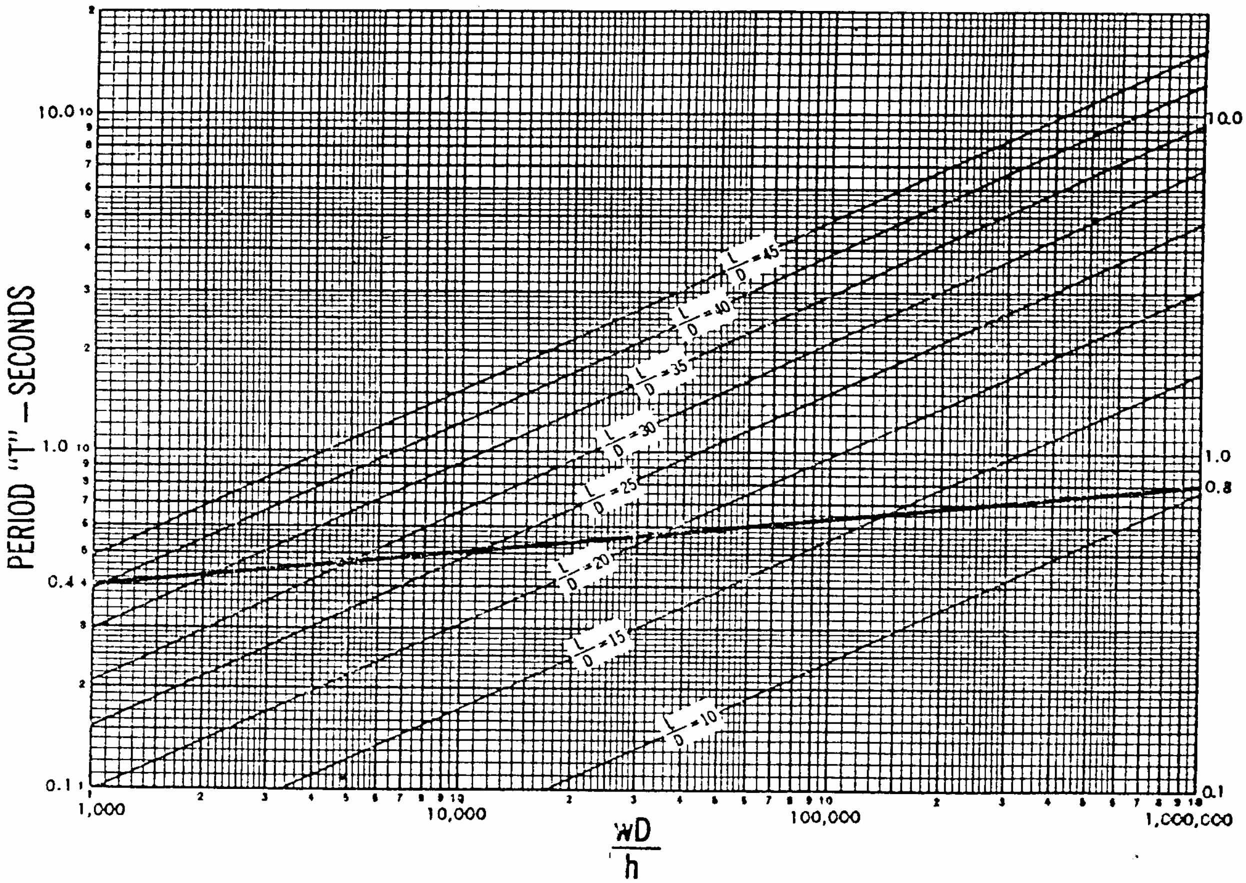

Fig. 1

applied only as a "yardstick" to provide reasonable protection with a minimum amount of additional material. However, anchor bolt stresses should be held low (15,000-16,000 psi) for these vessels or, if a higher stress is used, the design procedure outlined should be applied to them. Proper tightening of anchor bolts for vessels subject to dynamic behavior is of utmost importance and it is recommended that they should be pretightened to the predicted working stress to avoid stretching and loosening in service It is definitely unnecessary to apply this method to the design of the foundation unless the vessel is operating in a seismic area.

The design procedure just outlined produces consistent results and also provides additional material to resist the force due to mass-acceleration of the vessel in motion. A number of years ago, approximate calculations indicated that the total force due to wind load plus the force due to mass acceleration was about 1.5 to 1.70 times the static force due to 30 lb/ft, wind load for several different size vessels. It was found that the recommended design procedure resulted in shell thicknesses within a few thousandths of an inch of those obtained by the more lengthy approximation. Many critical vessels have been successfully installed which were designed to the seismic analogy method just described.

The same company that produced the first graph of equation (4) tentatively recommended the seismic design method using a 0.20 seismic factor for their vessels requiring dynamic design in order to be on the "safe side." Since this company was mainly interested in the response of vessels and other structures to earthquake induced vibration, they later revised their recommendations as follows:

Natural period of vibration	Earthquake coefficient
Less than 0.40 sec	0.20
0.40 sec to 1.0 sec	0.08 divided by period
Greater than 1 sec	0.08

Attention is again called to the fact that this paper is primarily concerned with vibrations induced by wind or other forces which occur more frequently than earthquakes and it should be noted that the vessel reported as Case II under Field Data is well within the later recommendations outlined here and vibration trouble was encountered. It is agreed that the current practice is probably adequate for earthquake design; however, all critical vessels (except the vessel reported as Case II) designed and installed by our company have been designed to the seismic analogy method using a 0.20 seismic factor.

Not all vessels designed as static structures have the same thickness of shell for their entire length and some vessels are of more than one diameter. These vessels, as well as many designed as dynamic structures cannot have their period of vibration estimated from the graph in Fig. 1 or equation (4). It is also desirable to know the change in the vibration period resulting from dynamic design. Of the several methods referred to in reference books on vibration [1, 2, 3] the numerical integration of the equation

$$T = 2\pi\left(\frac{\sum W y^2}{g \sum W y}\right) \tag{6}$$

is probably the easiest and safest method for the designer who is not a specialist in vibration to apply. This equation follows the Rayleigh method of approximation for finding the fundamental period of vibration as applied to a shaft or loaded beam on too supports. It will be shown that this equation is reasonably accurate for estimating the period of the first mode of vibration of vertical pressure vessels.

Equation (6) will result in an estimated period of vibration slightly lower than the actual period. The degree of accuracy is dependent upon the number of sections calculated in estimating the static deflections when the vessel is considered as a cantilever beam deflecting under its own weight. As an example, the period of vibration of a cylindrical shell 3 ft 0 in. diam by 3/4 in. thick by 90 ft 0 in. high was estimated under two separate conditions. In order to eliminate nonuniformly distributed masses, this shell was considered to have tray sections at one-foot intervals from the top to the ground and the heads were omitted. When calculated to equation (4), the period of vibration was found to be 1.088 sec. Dividing the shell into nine sections, each 10 ft 0 in. long and calculating the period to equation (6) resulted in an estimated period of 1.08 sec. which is 0.735 per cent low. On the other hand, when this same shell is divided into five sections having lengths of 30 ft 0 in., 20 ft 0 in., 15 ft 0 in., 15 ft 0 in., and 10 ft 0 in., the estimated period of vibration to equation (6) was 1.068 sec which is 1.84 per cent lower than the results from equation (4). Most vessels designed as dynamic structures have five to ten sections similar to the latter division and the weight is not always uniformly distributed. Field test have shown the calculated period of vibration to be 1.5 to 4.5 per cent lower than the observed periods for several different size vessels. This is in good agreement for large structures and it is reasonable to assume that the period of vibration obtained by the numerical integration of equation (6) will be approximately 5 per cent lower than the actual period .

Equation (6) is not difficult to integrate numerically, but care must be exercised to make certain that all factors affecting deflection are included. Instead of following a complete numerical integration, some designers prefer to estimate the deflections at the center of each section graphically by either the area-moment or conjugate beam method. The same results will be obtained. The choice of method depends upon the personal preference of the individual. An outline for the numerical integration of equation (6) when applied to vertical pressure vessels is given in the Appendix of this paper.

Discussion of Wind Effects

Tall, cylindrical structures such as pressure vessels and stacks are subject to being put in oscillatory motion by wind currents. The motion is at right angles or normal to the direction of the wind. This phenomenon is usually referred to as resulting from the Karman vortex trail (4, 5, 6, 10). The relationship between wind velocity and frequency of eddy shedding is given by the equation.

$$V = \frac{fD}{k} = \frac{D}{Tk} \tag{7}$$

Since we are primarily concerned with the resonant condition which occurs when the frequency of eddy shedding equals or is in the neighborhood of the natural frequency of vibration for the vessel the symbols for the vessel frequency and period of vibration are shown in equation (7). From this equation, we can estimate the critical wind velocity for most vessels.

The value for *k* was first determined in 1878 by V. Strouhal as 0.185 and is known as the Strouhal number [10]. It is assumed by some authorities to be within 0.18 to 0.27 and dependent upon the velocity of flow [4]. The reproduced graph shown in Fig. 2 gives the variation of the Strouhal number with the Reynolds number as obtained experimentally by Relf and Simmons [10]. Research engineers, employed by the same company as the author, reported the following values for *k* obtained from full size vessels after erection [12]:

k = 0.133, for a 7.67 O.D. insulated vessel at a wind velocity of 39.6 ft/sec (27 mph)
= 0.189, for a 3.0 O.D. vessel at a wind velocity of 32.25 ft/sec (22 mph)

The difference between the values reported from field data and the graph is probably due to the size of cylinders tested and the method of support. When the velocity of the wind is such that the frequency *f* in the equation corresponds to the natural frequency of the vessel, resonance occurs and the vessel will oscillate at an excessive amplitude. Since aerodynamic stability theory and calculation methods are beyond the scope of this paper, the reader should refer to Steinman's paper [10] and similar publications for additional information in this subject.

One vessel, not designed to the seismic analogy method outlined herein, gave trouble due to wind induced vibration. This vessel is identified as Case II under Field Data. It was found to be free from vibration when the wind was blowing from a direction such that nearby equipment disturbed the flow pattern and it is conceded by some individuals that the external attachments also helped to reduce or nullify the effect of periodic eddy shedding. It is recommended that any vessel, where possible vibration trouble is indicated, should have the external appurtenances located around its circumference and not placed on only one or two sides as was done with this vessel. The break in vertical ladder runs demanded by some states helps to accomplish this because intermediate platforms and ladders are distributed circumferentially. The same vessel which gave trouble when empty has been satisfactory after liquid loading. Therefore the additional damping effect of liquid loading cannot be ignored - on the other hand, neither can too much confidence be put in it as a cure-all.

Some engineers are also concerned regarding the possibility of the vessel being vibrated at a frequency corresponding to its second mode of vibration. The second mode of vibration for cantilever beam has a frequency of 6.37 times the frequency of the first mode [7]. This relationship will not necessarily hold true for multithickness and/or multidiameter vessels and more involved methods of analysis for the second mode frequency have to be employed. The Ritz method [8] which is a further development of Rayleigh's method can be used for these cases. It is sometimes referred to as the Rayleigh-Ritz method and should be applied by designers specializing in vibration problems. For the average vessel, it is not unreasonable to assume that the second mode frequency might occur between five to six times the frequency of the first mode. If a wind velocity of thirty miles an hour has been estimated to induce vibration in the first mode, it is reasonable to conclude that vibrations in the second mode will not be induced by any wind less than one hundred fifty miles per hour. On the other hand, a vessel subject to vibration in the first mode by winds of only ten miles an hour might be vibrated in the second mode by winds of fifty to sixty miles per hour if external attachments do not interfere with the periodic eddy shedding. Surrounding structures and terrain will also have some bearing on the considerations involved.

It is not the intention of this paper to overamplify the possibility of the second mode of vibration. Some engineers maintain that vibrations in the second mode could be catastrophic; how

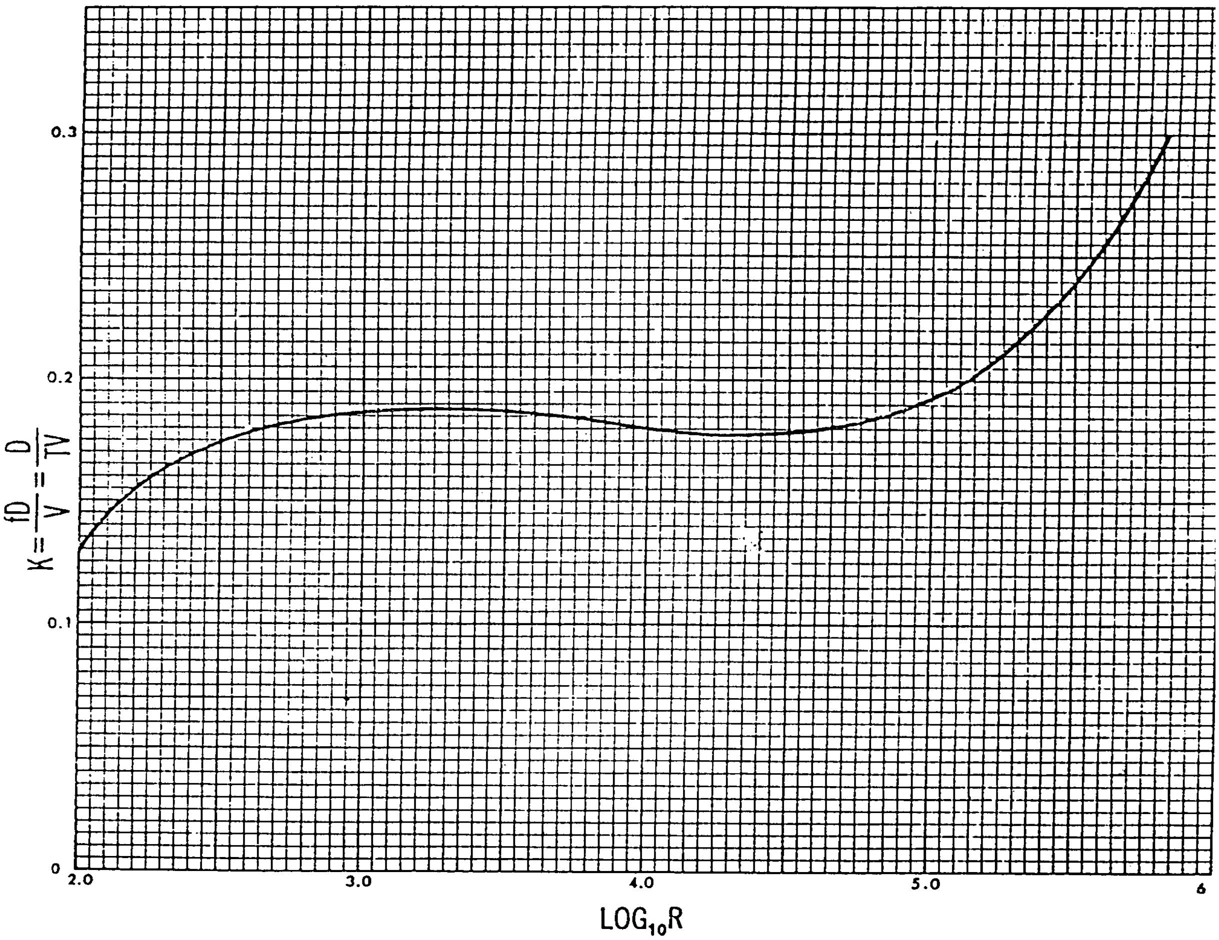

Fig. 2

ever, to the knowledge of this author no case of this type of failure has ever been recorded. In fact, no one has reported a second mode vibration in a self-supporting vertical pressure vessel and vessels are in service which have (L/D) ratios in the neighborhood of 40:1. Steinman's paper refers to the Meier-Windhorst tests at the Hydraulic Institute at Munich (1939) wherein the hydrodynamic oscillations of cylinders yielded sharply defined results for (a) the low velocity range, (b) critical range, and (c) high velocity range, and further states that, "In these vortex induced oscillations, there is no 'catastrophic range' of increasing amplification with unlimited increase of steam velocity" [13].

Discussion of Correction Methods

This paper would not be complete without a brief discussion of possible remedies if trouble occurs. One of the first things done to the vessel reported as Case II under Field Data was to try a spring loaded damping device originally designed by a large process engineering concern and shown in Fig. 3. This device had practically no effect on the behavior of the vessel. It can be argued that the spring load of 800 to 1000 lb results in a horizontal force of only about 200 to 300 lb at the top of the vessel and this small force will have very little effect on a moving vessel weighing 50,000 to 300,000 lb. If the spring load is doubled, the resisting load at the top of the vessel is still a small factor in reducing vibration or limiting the resulting deflection. However, if an accurate estimate can be made of the vertical expansion for the operating temperature involved, the spring could he designed and installed so that the assembly resulted in full cable tension during operation. This in turn becomes a hazard because of possible cable breaking which would endanger personnel. It has been suggested that the spring could be entirely eliminated, but this does not appear attractive due to the thermal expansion of the vessel during operation and the subsequent danger of cable breakage just outlined. Carrying the wires over sheaves and then straight down to the foundation as shown in Fig. 4 has similar drawbacks. Aerodynamic paneling similar to that used on the pipeline suspension bridge [6] has also been suggested, but is not always desirable from other standpoints, such as appearance and easy access to all sections of the vessel. If paneling is used, the sections should be attached by bolting them to clips which

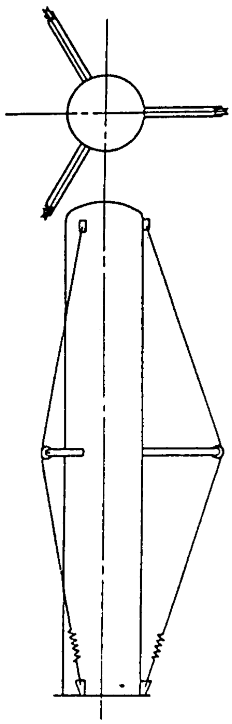

Fig. 3

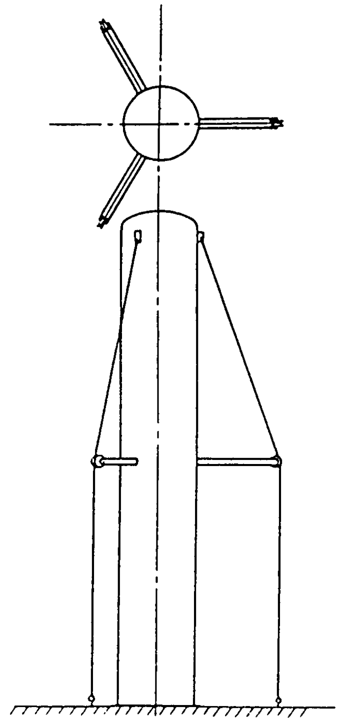

Fig. 4

have elongated or oversize holes to provide differential expansion during operation. In some cases additional rolled plate can be applied at the lower section which will increase the stiffness and lower the period of vibration. Vertical beams welded for the length of the vessel could be used, but are not recommended because of their restraining effect under thermal conditions and possible discontinuity stresses. One practical approach is to design the vessel so that there is a separate section in the top which can he partially filled with liquid (water or mercury if high density is required). The action of the liquid will rapidly dampen the vibration and help prevent excessive amplitude build-up, because at the instant the oscillatory motion has its maximum acceleration, the liquid is still moving in the opposite direction thereby creating a damping effect. This is the same effect (only of greater magnitude) as observed from the tray liquid reported in Case II under Field Data. Of course the choice of liquid and the possibility of using this type of damping is dependent upon the temperature involved. To date, it has not been necessary to resort to any of these methods for vessels designed to the method outlined in this paper.

Field Data

Case I 54 in. I.D. x 146 ft-0 in. High Vessel Shown in Fig. 5.

This vessel was designed to the seismic analogy method described in this paper. Field engineers checked the period of vibration by setting the vessel in motion and observing its frequency and amplitude with a surveyor's transit sighted on a target rod mounted horizontally at the top of the vessel. A stop watch was used to time the number of cycles. This vessel could be oscillated by two men exerting a back-and-forth motion at the top platform.

Whereas format data were not retained, the engineering records show that they found the period of vibration to be 1.67 sec. Calculating the period of vibration by the Rayleigh approximation (6), using six sections gives a period of vibration of 1.61 sec which is 3.6 per cent lower than the observed period.

Although the calculated wind velocity required to cause resonance is only 10-12 mph, this vessel has operated without any difficulty. The external attachments were well distributed about the circumference.

Case II 84 in. I.D. x 145 ft-6 in. high Vessel Shown in Fig. 6.

This vessel was not designed to the seismic analogy method. The thickness of the shell was increased in the lower section to withstand a high wind loading. During the construction period, this vessel was observed to be vibrating under certain wind conditions and, not only was the amplitude great enough to be alarming, but the anchor bolts stretched and an adjoining reboiler was loosened at its foundation.

Research engineers were sent to the field and made a comprehensive study of the installation. It was observed that resonance occurred at wind velocities in the neighborhood of 27 mph. As previously mentioned, critical vibration was induced when the wind came from a certain direction and, although the vessel could be mechanically vibrated from any direction, the vessel was "frequency-polarized" due to the orientation of the trays and welded downcomers. The maximum amplitude was 0.45 ft during resonance.

The vibration was recorded by strain gage-oscillograph equipment and accurate wind velocity readings were recorded at several different elevations. The recorded amplitude measurement was checked with a surveyor's target rod and transit as outlined under Case I.

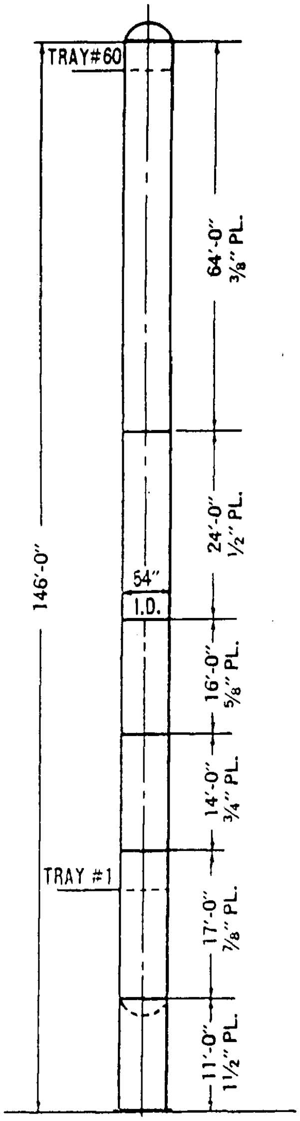

Fig. 5

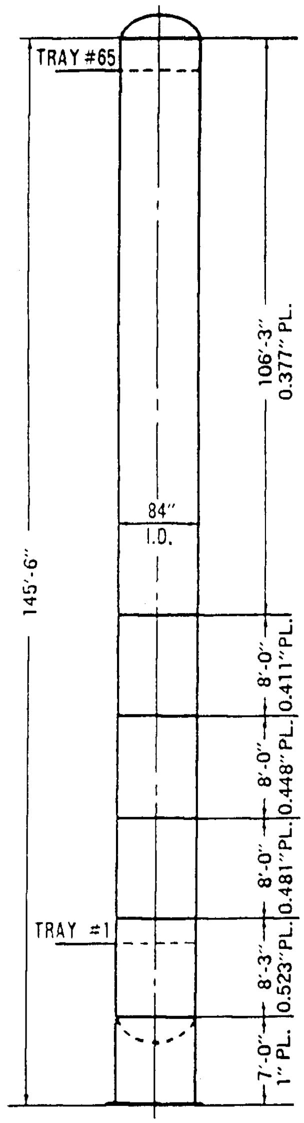

Fig. 6

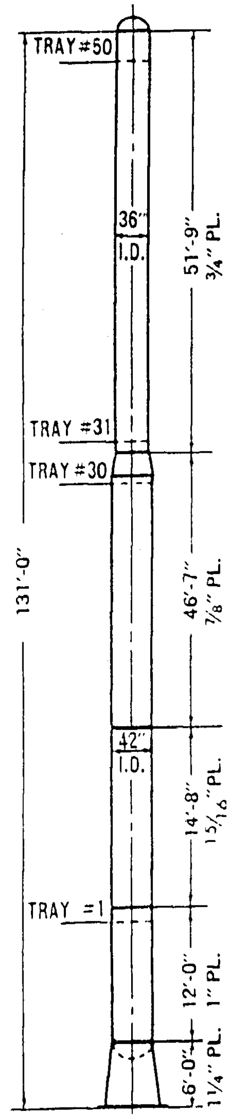

Fig. 7

During the investigation, the Strouhal number for this column (7.67 ft O.D. of insulation) and another 3 ft D column was obtained. These are reported in the Design Procedure section of this paper.

Using the term "per cent decrement," defined as each amplitude having a swing equal to a certain percentage less than that of its predecessor, this column was found to have a 31/2 per cent decrement when the vessel was empty and a 14 per cent decrement when the trays were liquid loaded

The logarithmic decrement for this column is approximately 0.035 without liquid loading and approximately 0.133 loaded with liquid.

The calculated period of vibration of 1.42 sec is 4.05 per cent lower than the observed period of 1.48 sec.

Case III 36 in. I.D. x 42 in I.D. x 131 ft-0 in. High Vessel Shown in Fig. 7.

This vessel, which was designed to the seismic analogy method, has a calculated period of vibration of 1.61 sec. Field readings were taken in the manner outlined under Case I. The readings shown in Table 1 were taken before the insulation was applied. The average observed period of vibration is 1.64 sec.

The readings shown in Table II were taken after the insulation was applied. The column was also pressurized at 210 psig and had a bottom temperature of 340 F and a top temperature of 90 F. There was no liquid on the trays but there was about four feed of liquid in the bottom. Both sets of readings were taken in still air. The average observed period is 1.69 sec.

The calculated period of vibration is only 1.83 per cent lower than the average from Table I and 4.15 per cent lower than the average from Table II.

The numerical integration of equation (6) consisted of considering the skirt to be 51 in. average diameter and the 42 in. D section as extending to the top of the conical reducer. In addition, the upper 51 ft 9 in. consisting of 3/4 in. thick plate was divided into three sections as was the 46 ft 7 in. of 7/8 in thick plate of the 42 in. D section, making a total number of 9 sections.

Calculations indicate a critical wind velocity in the neighborhood of 8-10 mph for this vessel. No excessive movement has

TABLE I

Observer	Time - Seconds	Number of Cycles	Amplitude at Start - Feet	Amplitude at Finish - Feet	Period - Seconds	Percent Decr't.
1	38	24	.15	.05	1.58	4.66
2	51.5	31	.20	.05	1.66	4.52
3	49	30	.20	.05	1.64	4.67
1	67	40	.20	.03	1.67	4.75
2	50.2	30	.20	.05	1.68	4.67
3	50.4	31	.20	.05	1.62	4.52
				Aver.→	1.64	4.63

TABLE II

Observer	Time - Seconds	Number of Cycles	Amplitude at Start - Feet	Amplitude at Finish - Feet	Period - Seconds	Percent Decr't.
3	31	19	.25	.05	1.63	8.55
1	33.2	20	.30	.05	1.66	9.00
3	38.2	22	.30	.05	1.74	8.18
1	38.2	22	.30	.05	1.74	8.18
				Aver.→	1.69	8.48

been reported and here again, the external attachments are well distributed about the circumference.

The increased per cent decrement in Table II is partly due to the addition of insulation which would have a more noticeable effect on a small diameter column. Internal pressure is believed to increase the stiffness and probably was a contributing factor. Whereas the bottom liquid is near the base and would not be expected to contribute to the increase in per cent decrement, it could conceivably have some effect.

The corresponding logarithmic decrements for the two conditions are 0.045 and 0.082.

Conclusion

1 Self-supporting vertical pressure vessels should always be investigated regarding their possible behavior under vibrating conditions.

2 If the statically designed vessel has a period of vibration such that it is necessary to consider it as a dynamic structure, it should be designed to the seismic analogy method using a 0.20 seismic factor. It is not necessary to apply this analogy to anchor bolts, *if they are not stressed over 15,000-16,000 psi.*

3 The period of vibration of multithickness vessels and most multidiameter vessels may be estimated by the numerical integration of equation (6). The length of the sections used for solving equation (6) should not exceed twenty to twenty five feet in order to have the estimated period within approximately 5 per cent of the true period. Complicated units with long, conical transitions (making it impractical to consider the cone as a straight shell having uniform properties) require more involved methods of approximation, or recognition that the estimated period for these units, if estimated to equation (6), may be more than 10 per cent in error.

4 The evaluation of wind velocity effects should include considerations pertaining to the distribution of external vessel attachments as well as the surrounding equipment and terrain. It should be borne in mind that liquid loading in vessels having trays will help dampen vibration, but should not be relied upon as a cure-all.

5 Anchor bolts must be properly pretightened to a torque which will prestress them an amount equal to their estimated working stress, otherwise they may stretch sufficiently to affect the period of vibration and possibly work loose.

6 If vibration trouble does occur, careful analysis of any proposed remedy must be made in order to avoid trouble from some other source.

Acknowledgment

The author wishes to acknowledge and express his appreciation for the co-operation of the Standard Oil Company of California, Salt Lake Refining Company, and the Research Division of The Fluor Corporation. The assistance of Mr. R. C. Baird as well as many individuals of The Fluor Corporation helped to make this paper possible and is gratefully acknowledged.

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10 D. B. Steinman, "Problems of Aerodynamic and Hydrodynamic Stability," Proceedings, Third Hydraulic Conference, June, 1946, Univ. of Iowa Bulletin No. 31, Page 139. Graph shown in Fig.2 reproduced by permission.
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13 Reference [10] pp. 144 and 145.

APPENDIX

I Estimating Period Of Vibration

The weights of the following items are used for estimating the period of vibration when applying equation (4) (or the corresponding graph of this equation) or when numerically integrating equation (6):

1 Weight of Shell and Heads.
2 Weight of Trays, Caps, and Internals.
3 Weight of Manways and Nozzles.
4 Weight of Insulation and Fireproofing.

The total weight (items 1 through 4) of the vessel or vessel section is considered as a uniformly distributed load acting on the vessel when it is considered as a cantilever beam, i.e., a cantilever beam deflecting under its own weight. Note that equation (4) requires the total weight to be divided by the total length in feet to obtain the Unit loading in pounds per foot, whereas equation (6) and the deflection equations shown in Fig. 9 are based on the total weight of each section under consideration.

The numerical integration of equation (6) is accomplished by dividing the vessel into the required number of sections - one section for each different thickness of plate with no section exceeding twenty or twenty-five feet in length, keeping in mind that the greater the number of sections, the more accurate will be the estimate.

After determining the weight and moment of inertia for each section, estimate the deflection at the e.g. of the section either

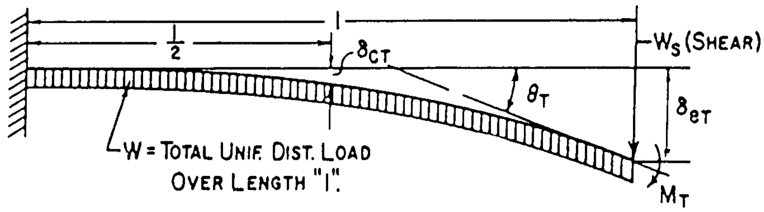


Fig. 8

TYPE OF LOADING	DEFL. AT c.g. (AT l/2) IN FT.	DEFL. AT END IN FT.	END SLOPE "θ" IN RADIANS = tan θ
(a) UNIF. DIST. LOAD "W"	$\delta_C = \frac{17Wl^3}{384E'I}$	$\delta_e = \frac{Wl^3}{8E'I}$ $2.825\,\delta_C$	$\theta = \frac{Wl^2}{6E'I}$
(b) END MOMENT	$\delta_{CM} = \frac{M_T l^2}{8E'I}$	$\delta_{eM} = \frac{M_T l^2}{2E'I}$ $= 4.0\,\delta_{CM}$	$\theta_M = \frac{M_T l}{E'I}$
(c) SHEAR	$\delta_{CS} = \frac{5W_S l^3}{48E'I}$	$\delta_{eS} = \frac{W_S l^3}{3E'I}$ $= 3.2\,\delta_{CS}$	$\theta_S = \frac{W_S l^2}{2E'I}$
TOTAL PER SECT.	$\delta_{CT} = \delta_C + \delta_{CM} + \delta_{CS}$	$\delta_{eT} = \delta_e + \delta_{eM} + \delta_{eS}$	$\theta_T = \theta + \theta_M + \theta_S$

Fig. 9

graphically or by the numerical procedure outlined herein. When applying the numerical procedure, it is necessary to find for each section (except the last one at the free end) the deflection at the center, the deflection at the end, and the end slope due to (*a*) the uniformly distributed load *W*, (*b*) the end moment, and (*c*) the shear load. These are found from the standard deflection equations shown in Fig. 9. The last section at the free end requires only the deflection at its center due to its own weight. Reference to Fig. 8 will immediately disclose that the last section on the free end does not have an end moment M_T or a shear load *Ws* to take into consideration and the end deflection and the end slope are not required to find the total deflection at the center. The center of gravity of each section is considered to be at its midpoint. Before proceeding with the deflection estimate, the designer should find all of the shear loads and end moments as shown in Fig. 10.

It is then a simple matter to find the total deflection of each section, square the deflection, and then tabulate the weight times the deflection and the weight times the deflection squared for each section, so that they may be added to find ΣWy and ΣWy^2 as shown in Fig. 11.

Equation (6) may be written

$$T = 1.108\left(\frac{\Sigma Wy^2}{\Sigma Wy}\right)^{1/2} \qquad (8)$$

to further simplify the arithmetic after the weight-deflection data are found.

As previously mentioned, many designers prefer to graphically estimate the deflection, but the numerical method is suggested for those who do not regularly use graphical methods. Actually either method requires about the same amount of arithmetic except those working graphically usually do not keep a detailed record of their areas and moments.

II Determination of Per Cent Decrement

Let X = amplitude of first swing.

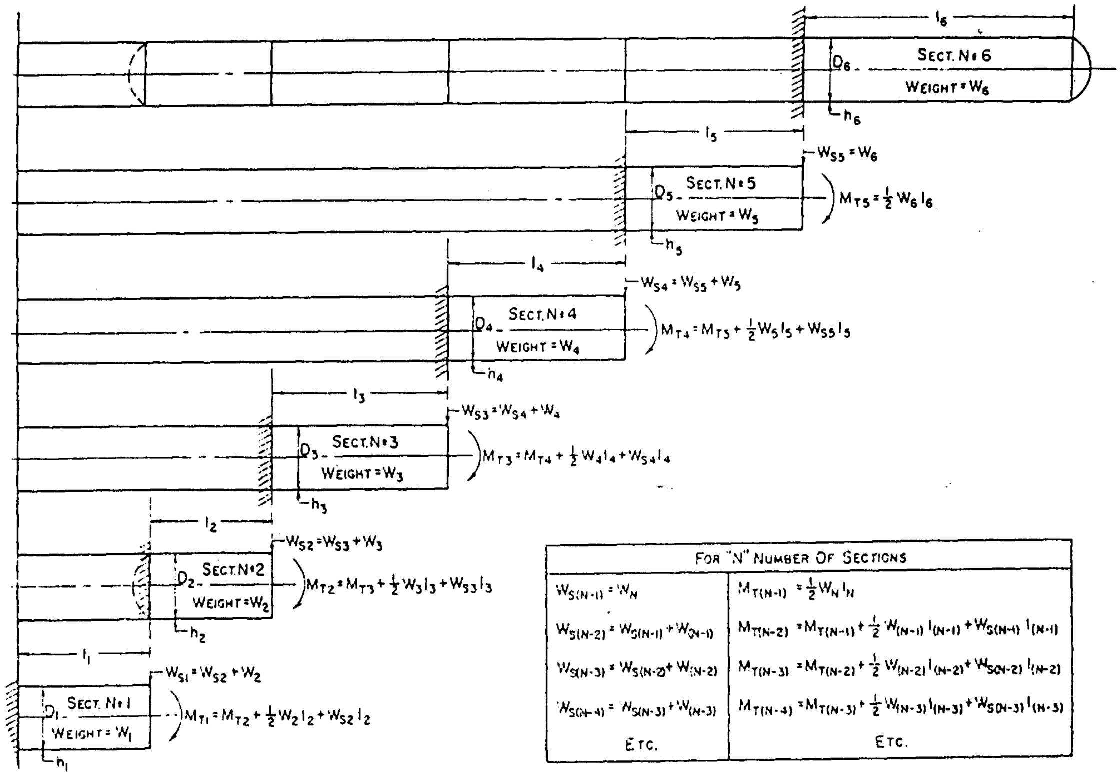
$$A = \left(1 - \frac{\%decrement}{100}\right)$$

Note: A five per cent decrement means that each swing is 5 per cent less than its predecessor.

Then:

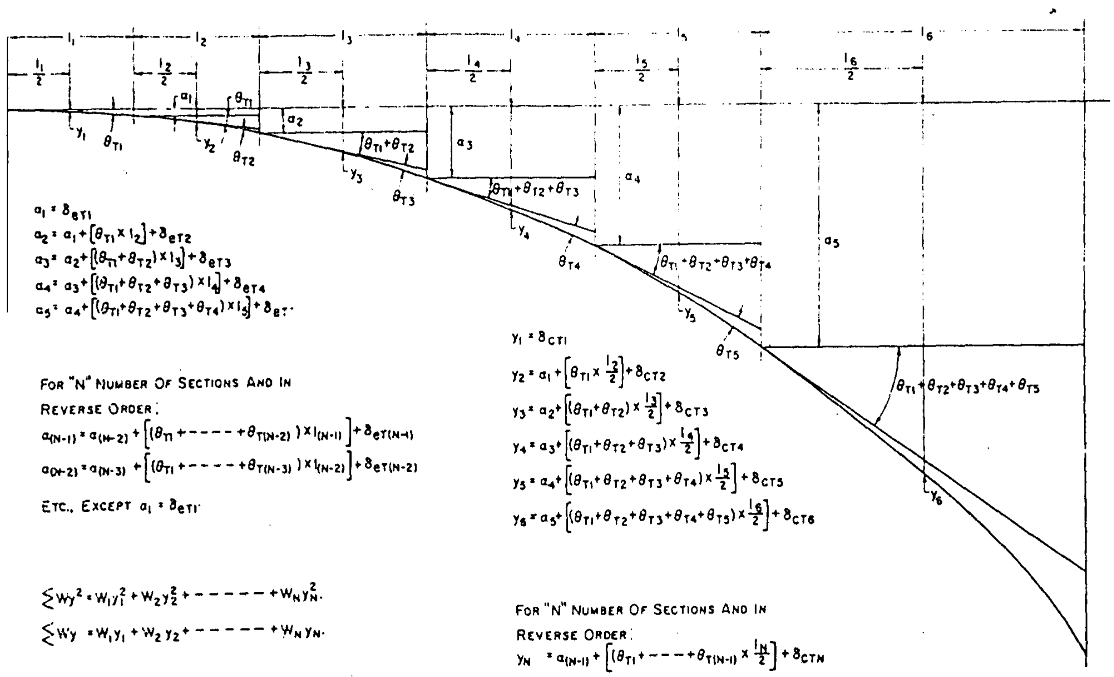
first swing amplitude = X
second swing amplitude = AX
third swing amplitude = A^2X
or
Nth swing amplitude = $A^{(N-1)}X$
and
per cent decrement = 100(1-A)

The term per cent decrement is of value when comparing the damping effect of different loading conditions for the same column.



For "N" Number Of Sections	
$W_{S(N-1)} = W_N$	$M_{T(N-1)} = \frac{1}{2} W_N l_N$
$W_{S(N-2)} = W_{S(N-1)} + W_{(N-1)}$	$M_{T(N-2)} = M_{T(N-1)} + \frac{1}{2} W_{(N-1)} l_{(N-1)} + W_{S(N-1)} l_{(N-1)}$
$W_{S(N-3)} = W_{S(N-2)} + W_{(N-2)}$	$M_{T(N-3)} = M_{T(N-2)} + \frac{1}{2} W_{(N-2)} l_{(N-2)} + W_{S(N-2)} l_{(N-2)}$
$W_{S(N-4)} = W_{S(N-3)} + W_{(N-3)}$	$M_{T(N-4)} = M_{T(N-3)} + \frac{1}{2} W_{(N-3)} l_{(N-3)} + W_{S(N-3)} l_{(N-3)}$
Etc.	Etc.

Fig. 10



$a_1 = \delta_{eT1}$

$a_2 = a_1 + [\theta_{T1} \times l_2] + \delta_{eT2}$

$a_3 = a_2 + [(\theta_{T1}+\theta_{T2}) \times l_3] + \delta_{eT3}$

$a_4 = a_3 + [(\theta_{T1}+\theta_{T2}+\theta_{T3}) \times l_4] + \delta_{eT4}$

$a_5 = a_4 + [(\theta_{T1}+\theta_{T2}+\theta_{T3}+\theta_{T4}) \times l_5] + \delta_{eT}$.

For "N" Number Of Sections And In Reverse Order:

$a_{(N-1)} = a_{(N-2)} + [(\theta_{T1} + ---- + \theta_{T(N-2)}) \times l_{(N-1)}] + \delta_{eT(N-1)}$

$a_{(N-2)} = a_{(N-3)} + [(\theta_{T1} + ---- + \theta_{T(N-3)}) \times l_{(N-2)}] + \delta_{eT(N-2)}$

Etc., Except $a_1 = \delta_{eT1}$.

$\sum Wy^2 = W_1 y_1^2 + W_2 y_2^2 + ------ + W_N y_N^2.$

$\sum Wy = W_1 y_1 + W_2 y_2 + ------ + W_N y_N.$

$y_1 = \delta_{CT1}$

$y_2 = a_1 + [\theta_{T1} \times \frac{l_2}{2}] + \delta_{CT2}$

$y_3 = a_2 + [(\theta_{T1}+\theta_{T2}) \times \frac{l_3}{2}] + \delta_{CT3}$

$y_4 = a_3 + [(\theta_{T1}+\theta_{T2}+\theta_{T3}) \times \frac{l_4}{2}] + \delta_{CT4}$

$y_5 = a_4 + [(\theta_{T1}+\theta_{T2}+\theta_{T3}+\theta_{T4}) \times \frac{l_5}{2}] + \delta_{CT5}$

$y_6 = a_5 + [(\theta_{T1}+\theta_{T2}+\theta_{T3}+\theta_{T4}+\theta_{T5}) \times \frac{l_6}{2}] + \delta_{CT6}$

For "N" Number Of Sections And In Reverse Order:

$y_N = a_{(N-1)} + [(\theta_{T1} + --- + \theta_{T(N-1)}) \times \frac{l_N}{2}] + \delta_{CTN}$

$y_{(N-1)} = a_{(N-2)} + [(\theta_{T1} + --- + \theta_{T(N-2)}) \times \frac{l_{(N-1)}}{2}] + \delta_{CT(N-1)}$

Etc., Except $y_1 = \delta_{CT1}$.

Fig. 11

DISCUSSION

M. Ludwig[2]

This paper deals with a problem that has long challenged this writer and his associates. His recommended procedure for calculation of dynamic wind forces can, however, lead to much more costly designs than we have found to be necessary. The author is primarily concerned with the possibility of forced resonant vibrations stimulated by transverse cyclic wind forces associated with the Karman vortex trail and, to deal with this possibility, suggests that flexible vertical pressure vessels be designed to resist a lateral force equal to 20 per cent of the gravity force. This lateral force is, for most flexible vessels (those with shell thicknesses greater than 0.4 in. if the total mass is twice the mass in the shell) greater than our customary design for either wind or earthquake. The justification for use of this seismic force is not presented, either as factor for a seismic design or for avoiding wind-induced vibrations.

An anomalous feature of the author's “seismic analogy” method of design is that it logically leads to the conclusion that a simple vertical cylindrical shell, such as a steel smokestack up to 0.8 in. thick, requires no special consideration because of possible wind vibration, whereas a fractionating column of the same thickness and diameter must be strengthened because of the added mass due to the insulation, trays and fluid thereon, ladders, piping, and other appurtenances. Such a conclusion cannot be supported by past experience; excessive vibration of steel stacks has occurred, whereas vibration of fractionating columns has seldom been a problem. The author notes one case of fractionating column vibration but this stopped when the column was put into operation; either the liquid on the trays provided adequate damping or the added mass of the liquid increased the natural period to a less critical value.

Our own experience is that the fractionating columns can be safely designed for static wind loads alone. The possibility of excessive wind vibration simply appears too remote to justify any added expense to prevent such vibration. Strengthening of the steel shell, by adding thickness, merely reduces the natural period of vibration and increase the wind velocity necessary to produce forced oscillations; it is not at all safe to assume that it would eliminate or reduce the amplitude of vibrations that might otherwise occur.

Why is it that tall vertical pressure vessels, such as fractionating columns, are far less severely affected by wind vibration than are self-supporting steel smokestacks? The greater mass per unit of gross cross-sectional area cannot alone be responsible since large above-ground oil pipelines have been observed to vibrate in the wind. There will be added aerodynarnic damping because of attached platforms, piping, etc., but it can be shown that the energy absorbed by this form of damping is probably not enough to limit the vibration amplitude to reasonable values. The external irregularities due to platforms, piping, etc., could, however, reduce the applied periodic wind force.

A sound theoretical analysis for forced vibration of the resonant frequency will answer the question raised in the previous paragraph. Actual numerical values for the possible vibration amplitude and resulting stress can be evaluated if the damping constant for the column can be determined or estimated. The detailed analysis, although straightforward, is too lengthy for inclusion in this brief review, but the final equations are listed. It is assumed that the vessel vibrates in a sustained wind as a uniform cantilever beam in the fundamental mode.

$$A_m = 3.30 \times 10^{-6} \frac{C}{S^2 In\Delta} \frac{D^2}{Bt} \qquad (9)$$

$$\sigma = 1.76\, E \frac{D\, A_m}{L^2} \qquad (10)$$

or

$$\sigma = 5.81 \times 10^{-6} E \frac{C}{S^2 In\Delta} \frac{D^3}{BtL^2} \qquad (11)$$

in which

A_m = maximum vibration amplitude at top of column
B = ratio of total mass per foot to mass per foot of steel shell
C = coefficient for peak periodic wind force - given as 1.71 by Steinman (reference [10] of the paper)
D = diameter of shell
E = modulus of elasticity of steel
L = height of column
S = Strouhal number
t = thickness of shell
Δ = amplitude ratio for two successive maxima, for free oscillations
$\text{In}\Delta$ = natural logarithm of Δ. (This is the "logarithmic decrement." The "damping ratio" or ratio of actual to critical damping is equal to the logarithmic decrement divided by 2π)
σ = peak vibratory stress at base of column

Any consistent set of units may be used in these equations.

As a numerical example, let

C = 1.71
S = 0.20
B = 2.00
D = 4.00 ft
L = 150 ft
t = 1 in. or 1/12 ft
E = 30×10^6 psi or 4320×10^4 psf
$\text{In}\Delta = 0.10$ (damping ratio = $0.10/2\pi = 0.0159$)

Then, from equation (9), A_m = 0.135 ft or 1.62 in.
From equation (l0) or (11), σ = 1270 psi.

The possible deflection and stress calculated here are hardly large enough for concern. The assumed damping is reasonably small but could be much less without serious results. As a matter of further interest, the natural period of vibration for this column is 2.39 sec, the wind velocity for resonant vibrations is 8.4 fps, and the Reynolds number for this wind velocity is 214,000. This is within the region where C should be around 1.71 and S around 0.20, as was assumed. A value of 1.71 for C is probably the maximum obtainable. It actually determines the peak value of the periodic force, which probably includes higher harmonics; the coefficient for the fundamental frequency component may be substantially less. Also the coefficient will be less if the column is not a true circular cylinder or if the Reynolds number is greater than 500,000.

Note especially, as shown by equation (11), that the bending stress is, for a given damping ratio, proportional to the cube of the diameter, inversely proportional to the square of the height, inversely proportional to the shell thickness, and inversely proportional to the mass ratio B. The greater values of the thickness and mass ratio for pressure vessels, as compared to steel smokestacks, are particularly important in minimizing the seriousness of possible wind-excited vibration. Thus, both experience and theory lead to the conclusion that vertical pressure vessels, such as fractionating columns, are not nearly as likely to vibrate excessively in the wind as are self-supporting steel smokestacks.

[2]Standard Oil of California, San Francisco, Calif.

Earl J. Hicks[3] and J. R. Sellers[4]

The author implies that the vibration of a tall vessel is a case of forced vibration, with resonance occurring when the Karman vortex trail frequency corresponds with the natural frequency of the vessel. This is supported by Baird [14][5] in. his investigation of a pipeline bridge vibration. We have had a similar experience with a pipeline bridge in which a critical wind velocity was observed. A search of the literature finds an exception to this with experimental data to support the claim that tall stacks vibrate as self-excited vibrational systems with no critical wind velocity causing forced vibrational response. Ozker and Smith [15] make the following statements in the summary and conclusions portion of their paper which are contrary to this paper: (1) "The stack structure under wind action constitutes a self--excited vibration system": (2) "The vibrational frequency is the natural frequency of the structure and remains constant for all wind velocities": (3) "The stack is at resonance at all times"; (4) "There is no critical wind velocity in the sense of forced vibrational response"; and (5) "The amplitude increases with increasing wind velocities." If this were found to be true for a stack, could it not also be true for a tall vessel? The bulk of the data seems to support the theory of forced vibration excited by the Karman vortex trail, but there is this one notable exception. There may be others.

The basic assumption in calculating the natural frequency of a tall cylindrical vessel is that the base is fixed at the top of the foundation. This implies that the horizontal displacement or deflection is due to the elastic deformation of the vessel and that the horizontal displacement due to the elastic deformation of the soil is negligible. This means that the vessel would act like a true cantilever beam fixed at one end. This is one extreme assumption.

The opposition extreme assumption would be that the vessel and foundation is a rigid structure resting on an elastic subgrade. This implies that the horizontal displacements of the tower are negligible compared to the deflection due to deformation of the soil.

Assuming the latter assumption to be correct, it can be shown for a vessel on an octagonal foundation resting on an elastic subgrade that the natural frequency of vibration is equal to the following:

$$f = 0.232\frac{B}{H}\left(\frac{d_s}{q}\right)^{1/2}$$

where

- B = short diameter of the base in feet
- H = distance between the base and the center of gravity of the foundation and vessel
- d_s = coefficient of dynamic subgrade reaction
- q = static soil pressure per unit of area

From this equation the following conclusions can be drawn. The softer the supporting soil, the lower is the natural frequency and the frequency may be raised by increasing the area if the foundation base [16].

In general, observed frequencies have been lower than the calculated frequencies. This is probably due to the vessel and foundation acting together in a manner somewhere in between the two extreme assumptions. If the supporting soil is relatively strong, the first assumption is more nearly correct. If the supporting soil is relatively soft, the actual condition tends to approach the second assumption. Past observations on various stacks have indicated reasonably close agreement between the observed frequency and the calculated frequency, the latter being based on the fixed cantilever beam assumption. If the frequency of a vessel is determined, assuming a fixed cantilever beam, and the supporting soil is soft, the error would be on the unsafe side; that is, the calculated critical wind velocity would be high.

The method of design suggested for wind induced vibration appears to be arbitrary and most likely finds justification in the number of successful towers. It would be desirable to have a more analytical approach to the problem. Apparently, any relation between the wind induced vibration and the 0.2 seismic factor is purely coincidental.

It appears a more realistic approach would be to equate the energy input in terms of amplitude to the energy dissipated in terms of amplitude and solving for the amplitude where the energy input is equal to the energy dissipated. The vibrations at this point would be an undamped steady-state free vibration and the resulting amplitude would be a maximum. Knowing the maximum deflection, the maximum stress could be readily calculated. If the resulting stresses are excessive, then other methods to dissipate the energy would have to be used.

This method would require reasonably accurate knowledge of the coefficient of lift C_L, the Strouhal number S, and damping decrement σ. Only meager information on the numerical value of these coefficients is available at this time. It appears desirable to study the loads and dynamic response in a series of wind tunnel tests. Does the author know of any such studies?

Aerodynamic paneling, liquid loading on trays, and liquid chambers are practical solutions. One other worth mentioning is that of tieing adjacent vessels and structures together. Vessels arranged in a triangle or square with common ties have a different vibrational mode and a higher resonant frequency than a single tower. In addition, more damping is introduced. A single vessel tied at an immediate level to an adjacent structure will vibrate in a different mode and higher frequency. These can be used as safeguards for vessels claimed as critical.

The liquid chamber suggested is one form of Frahm dynamic absorber system.. A word of caution should be given on this application. The Frahm dynamic absorber system has two resonant frequencies and one frequency of zero amplitude. A properly sized liquid chamber would decrease the amplitude to zero at one frequency, reduce it over a limited range of frequencies, but amplify it at two frequencies outside this range. In other words, at certain frequencies the liquid may not slosh to oppose vibration, but instead be in phase to amplify it. Frahm antiroll tanks for ships are dealt with in Den Hartog's book [17]. A liquid chamber appears to be an energy absorber of this same type.

In Case III, no excessive movement has been reported although the critical wind velocity is in the neighborhood of 8-10 miles per hour. In Case I, the vessel has operated successfully with a calculated wind velocity to cause resonance of 10-12 miles per hour. Both of these vessels were designed to the seismic analogy method. In Case II where the seismic analogy method of the design was not used, the vessel vibrated during the construction period. Does the author imply that since Case III and I were dynamically designed and Case II was not that this was the main reason that vibration did not occur or could it be that the ladders and platforms were distributed around the former two vessels such that the formation of the Karman vortex trail does not materialize? It appears that the ladders, platforms, piping, and insulation might be the major factor in preventing vessels from vibrating. No doubt there are many vessels with a height to diameter ratio greater than 20 which are operating

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[4] Engineering Department, Phillips Petroleum Company.
[5] Numbers in brackets designate References at end of this discussion.

satisfactorily and have not been designed as recommended by the author. The author implies that any vessel with a period which falls above the line shown in Fig. I should be designed dynamically. Would this be true for vessels whose heights are less than 100 or 75 or 50 ft? In general, is there some approximate height regardless of the H/D ratio where vessels below this height would not have to be investigated?

Although the author does not say specifically that the vessel should be designed such that the "critical wind velocity" is high enough to exclude the possibility of the second mode of vibrations it is mentioned. If this item is not critical, then why is it necessary to use such an accurate and time consuming method to determine the period when it could probably be estimated by other methods. Apparently, the designers were not concerned in Cases I and III about the low critical wind velocity since they were designed dynamically.

The author's paper is very timely indeed and is viewed with considerable interest since the trend in pressure vessel design is to increase their height and decrease the internal pressure. We wish to compliment him on a very excellent paper and also thank him for stimulating our interest in the subject.

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L. Acquaviva[6]

The proposed criteria for dynamic design of pressure vessels appears to yield more conservative results than the method we have been using for many years. It appears that the seismic coefficient of 0.20 applied on all vessels with height to diameter ratio exceeding 15 or natural period exceeding 1.0 sec may be too conservative based on our experience. For example, as an extreme case, there is a tower within the Esso interests which is 173 ft high., topmost 40 per cent of height is 6 ft 6 in. in diameter and lower 60 per cent 5 ft 0 in. in diameter. The approximate L/D ratio is 30, natural period of vibration 3.7 sec and critical wind velocity 6.5 miles per hour for the top section. The tower is on pile foundations and has given no vibration difficulties even though subjected many times to the critical wind velocity for the top section; nor has there been any tendency to vibrate at its second mode for which the critical wind velocity is about 40 miles per hour. The bottom section shell thickness for this vessel is 7/8 in. The thickness required to conform with proposed dynamic design would be 1 3/8 in.

Unlike steel stacks, for which there are many instances of excessive vibration reported in technical literature, Case II in the paper is the first example to our knowledge of such occurrence in vertical pressure vessels. In this case, the maximum amplitude of 0.45 ft during resonance does not appear to be of sufficient magnitude to cause excessive stresses in the vessel shell. It would be of interest to know if an remedial measures were taken in this case.

[6] Esso Research and Engineering Company, Linden, N. J.

Donald J. Bergman[7]

This paper is particularly interesting because it includes some actual field data which have been extremely difficult to get. It seems important to note that minor changes in resonant frequency have little effect on the over-all situation as this merely results in a slightly different wind velocity to reach resonance. It was much more important to note how greatly the liquid on the trays of a column increased the damping effect. This absorbs the energy of the wind forces and cuts down the magnification factor, decreasing the maximum. amplitude of vibration.

We had several instances of stack vibration starting in 1938, all far removed and all solved by use of permanent guys. The first key to the causes came from Sir James Jeans’ book "Science and Music." Here the comment was made that a set of eddies broke off from a cylindrical surface every 5.4 diam along the wind stream and that these eddies caused a push from side to side. The example was given of a ship with 1/2 in. rigging at sea in a 40-mile gale where the frequency comes out to correspond to middle "C" on a piano.

The policy of providing light guys and spreaders was adopted for columns with high L/D ratios and springs were provided to take care of column elongation by heat. Several hundred columns were thus equipped over past years without having one case of resonant vibration. Perhaps this was just fortuitous because of presence of ladders, platforms, etc., to interfere with resonance, and absence of critical winds. Most of these columns were within the critical range as defined by the author.

Later study indicated the same conclusions that the author reached: That the guys and springs were probably not effective and, consequently, about a year ago provision of these guys was abandoned. Since then one tower was found to have a resonant period with the wind and permanent guys were added.

Perhaps it should be noted that the elasticity of fixed guys actually makes them springs with a very high constant. Limiting the movement of the tower is the desired result, and this the guys do by resisting the wind forces.

The lateral forces resulting from the action of the Karman eddies seem to be somewhat greater than the normal wind drag for any wind velocity. With a low decrement, or high magnification, say, 20, only a relatively small force, 1 lb/sq ft, is required to give the deflection obtained when designing for a 100-mile per hour wind at 20 lb/sq ft, on the projected area of the column.

The lateral forces generated on cylinders were actually utilized shortly after World War I by the German *Flettner* rotor ship which cruised from Germany to New York using two 10 x 60-ft cylinders mounted on the deck and rotated in order to furnish a fixed instead of an oscillating force.

Rouse’s book “ Engineering Hydraulics” on page 130 gives formulas for an approximation of the aerodynamic lift caused by the Karman eddies on fixed and rotating cylinders. Additional information for higher Reynolds numbers was obtained from Prof. L. Landweber of the Iowa Institute of Hydraulic Research by correspondence. For the 7.67-ft column which gave trouble with a 27-mph wind, I calculate a loading of 9.1 lb/ft or 1.18 lb/sq ft of projected area. Stiffening a column and increasing its frequency will decrease the deflection for a given load, but the wind forces increase as the square of the wind velocity.

Some comment was made regarding the second mode of vibration. This will always require a very much higher wind velocity than the first mode, but the column is so much more stiff because of the complicated second mode curve that it is probable that the vibration is not noticeable if it does occur.

[7] Engineering and Development Department. Universal Oil Products Company, Des Plaines, III. Mem. ASME.

Patent no. 2604838, dated July 29, 1952, to W. B. Traver, of Standard Oil Company of Indiana, teaches provision of a roughened surface near the top of a stack either by small strips ranging from 1/4 to 1 in. depth, or by the use of steel grating. This increases the skin resistance on the surface of the cylinder to prevent formation of Karman eddies. However, it does not seem to be a very practical remedy because of the cost of attaching the large number of small strips to the stack.

Author's Closure

Before replying to specific questions or comments, the author wishes to thank the discussers for their review and comments pertaining to the design of vertical vessels subject to vibration.

It is hoped that sufficient interest has been stimulated in this subject to result in obtaining more field data than we presently have at our disposal. Up to the present time very little has been done to obtain information pertaining to tall, slender vessels. If no complaint was received from the operators, it has been assumed that no critical vibration would ever occur. This is a normal and expected attitude; however, it has not contributed to our knowledge of vessel behavior as heights have increased.

As pointed out in this paper and described more in detail in the paper given by Mr. Baird, we have had definite experience with one vessel which did give trouble and, after completing this paper the author was informed that this vessel still has to have anchor bolts retightened at intervals which indicates that some excessive vibration may still be taking place.

The author does not agree with Mr. Ludwig that we can generalize regarding the relationship of total mass to the mass of the shell itself. Mr. Ludwig's use of a mass equal to twice the mass of the shell in his 4-ft 0-in. diam X l-in. thick example would result in very heavy internals. To me, each vessel should be considered individually during the design stage. Obviously, there will be those where conditions of terrain and prevailing winds will cast doubt regarding the justification of added cost to the vessel.

The cost angle of designing to the method recommended in this paper has been overemphasized. Taking Mr. Ludwig's example and designing it to the 0.2 seismic analogy results in the following thicknesses:

Top	104 ft	—	1-in. plate (orig. thickness)
Next	16 ft	—	1 1/8-in. plate
Next	8 ft	—	1 1/4-in. plate
Next	8 ft	—	1 3/8-in. plate
Next	8 ft	—	1 7/16-in. plate
Skirt	6 ft	—	1 7/16-in. plate

This represents an increase of only 7,500 lb added to a vessel originally designed as 146,000 lb (when l6,000 lb is subtracted for insulation). This approximation is based on ASTM A-212-B Material, S.R. and x-rayed, and includes 1/8-in. corrosion allowance in the l-in. plate, which means that this vessel would have 7/8-in. thickness for resisting pressure in the hoop direction. We then have 7/16 in. of material available to resist the bending in the longitudinal direction. Mr. Ludwig ignores the pressure stress when he reports his bending stress due to his estimate of the maximum amplitude of vibration. Costwise, this vessel would represent an investment of approximately $36,500 as originally outlined. The added material and labor would be approximately $1350 which is an increase of only 3.7 per cent. In this case, the added material accomplishes two things; first, it increases the section of the vessel where bending stresses occur and will reduce the possibility of failure from fatigue, a condition which could exist if the critical wind velocity was prevalent; second, it gives more resistance to bending if a harmonic condition occurs.

Mr. Ludwig reported a critical wind velocity of 8.4 fps or 5.7 mph, and apparently used a 4-ft 0-in. diam with his selected Strouhal number of 0.20. Adding 3 in. of insulation, and using a Strouhal number of 0.18 (based on field data for two vessels reported in the paper) gives a critical velocity of 7.1 mph for the vessel as originally outlined and 7.8 mph for the design as recommended in the paper. (The period of vibration decreased from 2.39 to 2.19 sec.) It is agreed that the recommended design does not materially affect the critical wind velocity for most vessels.

It is also agreed that stacks may cause more trouble from vibration than vessels, because they do not have liquid loading and are usually lighter in weight. However, the inference pertaining to ignoring a stack of 0.8 in. thick should be further clarified, because the *L/D* ratio would determine whether or not the stack should be designed for vibration. Also, there is no mention of stacks in this paper and it is *not* recommended that this design procedure should be applied to them. Since processing is not involved, stack designers usually increase the resistance to vibration by a generous conical section in the lower zone which sometimes runs from ¼ to 1/3 the height of the stack. The pressure vessel designer can seldom resort to this because of internal construction. In order to clarify the statements concerning stacks, the following tabulation is based on Mr. Ludwig's definition of a flexible vessel; i.e., one with a shell thickness greater than 0.4 in. if the total mass is twice the mass in the shell:

Diam, in.	Shell th, in.	Vessels wt/ft,[a] lb	L/D	Max L, ft
36	7/16	360	20	60
48	7/16	480	18	72
60	7/16	600	17	85
72	7/16	720	15	90

[a]This is double the wt/ft for 7/16-in. plate.

Diam, in.	Shell th,[b] in.	Vessels wt/ft, lb	L/D	Max L, ft
36	7/16	180	24	72
48	7/16	240	21	72
60	7/16	300	19	84
72	7/16	360	18	108

[b]*L/D* and max *L* values for stacks actually are independent of shell thickness because the value of the abscissa (*wD/h*) of the graph shown in Fig. 1 does not change as the material thickness is changed (*w* and *h* are proportional).

A comparison will show that the 36-in-diam vessel can be considered as a static structure if not over 60 ft high, whereas the 36-in. stack could be considered as a static structure up to 72 ft high, regardless of its thickness. Similar values are shown for other diameters. As previously mentioned, it was not intended to apply the graph shown in Fig. 1 to stacks. However, there does not appear to be a great discrepancy in this respect when it is realized that all of the metal put in the stack may be used for structural strength since there is no internal pressure.

Stacks are frequently built using the thinnest calculated material thickness, and sometimes local buckling has occurred due to warpage and/or external forces. I do not believe a true comparison between stacks and vessels can be made.

The calculation of the periodic force and resulting amplitude is difficult to make with any degree of accuracy, because we lack field data on vessels. Past attempts to apply some of the data from Reference [10] in the paper have not always give results consistent with the field data taken to date. This does not mean the approximations of this nature should not be made, but we cannot place too much reliance in them.

Mr. Ludwig's bending stresses are very low because he included all of the shell material for bending, whereas in actual design work we have to combine the pressure stresses with the bending stresses. Also, Mr. Ludwig's stress is based on his estimate of the amplitude and ignores the fact that a deflection exceeding the normal amplitude might result if resonance occurs, as reported for the vessel outlined as Case II and reported by Baird.

Mr. Hicks and Mr. Sellers discuss the possibility of the foundation and soil-bearing capacity as contributing to the flexibility of the system. This possibility has not been ignored. However, there has been no observation to justify this assumption. A check into the effect of anchor bolt stretch did not affect the period of vibration estimate sufficiently to bother with it—other than to require pretightening to reduce elongation during vessel deflection. When the total mass of concrete and the surface charge of earth is considered, it is difficult to agree that they materially influence the natural frequency. If the soil bearing capacity is low, it is usually necessary to drive piles to support vessels and similar heavy equipment.

The desire for analytical approach is appreciated. However, pressure vessels do not always lend themselves to a true scientific analysis. In this case, so many variables exist that assumptions made in order to solve equations are apt to produce misleading results. The fact that a number of successful vessels have been built using this sornewhat empirical approach is reasonable justification for its consideration. If we cannot justify a design method on the basis of successful operation, then we would be forced to discard many of our practices which are based on experience, including earthquake design. It is also noted that some reviewers are equally positive that it is unnecessary to take any precautionary measures because they have not experienced any difficulty in the past. Therefore, we have one more instance where the variables are too many to draw a definite conclusion and data pertaining to location, terrain, wind currents, detail vessel design, and vibration for each case are not available. If we had at our disposal sufficient data taken from existing units, we probably could work out a more analytical approach.

To my knowledge there have been no wind-tunnel tests pertaining to vertical vessels except those mentioned in Reference [10]. Investigation of testing models for tall, slender columns will immediately reveal that, in order to obtain reliable data for *L/D* ratios of 30 and 40 to one, the model size will be difficult to work with, because the diameter is so small in order to avoid excessive height. Some of the larger wind tunnels could probably handle models large enough to be practical, but the cost would be very high.

Connecting critical vessels to nearby structures of to adjacent vessels is always desirable if the location of the equipment permits. This cannot be accomplished in many instances.

I do not agree that the use of a single liquid chamber is comparable to Frahm antiroll tanks. The type of motion differs and I believe that there is very little possibility of the liquid amplifying vibration. However, as pointed out in Conclusion No. 6 in the paper, any proposed remedy must be carefully analyzed to avoid additional trouble from some other source.

The paper does imply that the vessel reported under Case II might have been satisfactory if designed to the seismic analogy method, but there is no proof of this. Recognition is given to the desirability of distributing ladders, platforms, and other accessories circumferentially about the vessel to reduce the effect of periodic eddy shedding.

The heavy reference line shown in Fig. 1 is independent of height and considers *L/D* ratios more important than height. A vessel 30 in. in diameter and 90 It 0 in. high can be more critical than a 200-ft 0-in-high vessel of a larger diameter. This reference line from 0.4 sec on the left to the 0.8 sec on the right of the graph is empirical. There certainly are vessels having periods of vibration above this line which were not designed as recommended that are satisfactory. This line at one time was a horizontal line at 0.4 sec, and experience indicated that it could be safely changed as shown. Future data may result in another revision for this limit.

The method outlined in the paper for numerically estimating the period of vibration for a vessel having a shell varying in thickness is intended for designers who are not experts in vibration. We are programming this work for computing machines along with our other vessel program, but computing machines are not always available, and other methods of approximation are not very reliable unless performed by experts in the field of vibration. It is not always necessary to estimate this period and frequently it can be approximated by using the graph in Fig. 1. It is up to the designer to decide the degree of accuracy he wishes to attain.

The column described by Mr. Acquaviva has considerable difference in the diameter of the top and bottom section. Our experience has been that vessels of this type are not as critical as tall, slender columns which are the same diameter for their entire length. I have some reservation regarding the accuracy of estimating critical wind velocities for this type of vessel when the diameter difference exceeds 6 to 12 in.

Here is another example of experience and each individual will have his own idea of :a correct solution.

If I were compiling data pertaining to existing vessels, I would definitely attempt to include all possible information on surrounding structures, wind currents, and geological data, in addition to details of vessel construction. A successful installation in one locality may not be trouble free in another.

The vibration dampener shown in Fig. .3 in the paper was applied to the vessel which vibrated at an excessive amplitude. This had practically no effect on the behavior of the vessel, but was not removed. Liquid loading helped to prevent excessive amplitude build-up, but as previously mentioned, it was recently discovered that the anchor bolts have to he retightened at frequent intervals.

My reply regarding cost of designing to a 0.2 - seismic factor was included with the reply to Mr. Ludwig's comments.

Mr. Fred Ruud's statement that the U. S. Bureau of Reclamation uses a seismic analogy method for some of their tall vessels is particularly interesting in view of the fact that seismic factors of 0.25 to 0.30 were mentioned, and the author wishes to thank Mr. Ruud for his interest and express regret that he did not have the opportunity to prepare written discussion on this paper.