APPENDIX

Flexibility Analysis of Piping Tank Combination —Example Problem

In order to illustrate the interrelationship between a piping system and the connecting tank nozzle, the following idealized two-dimensional example problem is presented. The more general form of flexibility equations for a piping system in a space connected to a tank can be found in reference [1].

Assume end 0 in Fig. 6 is to be fully anchored. The compatibility equations of this piping system considering the elastic restraint of the tank can be expressed in the form

$$\left(S_{aa} + \frac{EI}{K_R}\right) F_R - (S_{ab})F_L + \left(S_b - \frac{EIL}{K_L}\right) M_L = EI\{\Delta R_{OP} - \Delta R_{OS}\} \quad (4)$$

$$-(S_{ab})F_R + (S_{aa})F_L - (S_a)M_L = EI\{\Delta L_{OP}\} \quad (5)$$

$$\left(S_b - \frac{EI}{LK_R}\right) F_R - (S_a)F_L + \left(S + \frac{EI}{K_L}\right) M_L = EI\{\theta_{LOP} - \theta_{LOS}\} \quad (6)$$

S_a, S_b, S_{ab} are the summed shape coefficients of the piping system as determined by the procedure outlined in reference [1]. F_R, F_L, M_L are the redundant radial and vertical force and vertical moment at point D. EI is the flexural rigidity of the attached piping.

ΔR_{OP}, ΔL_{OP} are the unrestrained expansions and θ_{LOP} is the unrestrained rotation of the piping at point D. ΔR_{OS}, θ_{LOS} are the outward growth in the radial direction and downward rotation in the meridional direction of the tank shell-nozzle connection due to thermal and pressure loading only. These deformations are furnished by the tank designer or owner to the piping designer.

Solving these three simultaneous equations yields the values of F_R, F_L, and M_L. The deformation of the tank shell due to the reaction F_L is neglected as it is assumed that the tank shell-nozzle connection is extremely rigid in the longitudinal direction. The effect of very large upward F_L loads would be to pick up a large portion of the tank shell and a portion of the bottom. Conversely a large downward F_L load would add load to a portion of the foundation.

Example Problem. The following problem exemplifies the use of the curves in calculating the local translational and flexural rigidity of a tank-nozzle connection. The calculated values can be incorporated into the analysis of the piping system as shown.

Details of Flat-Bottom Tank:
Cylindrical tank diameter = 180 ft
Height of stored product = 70 ft. (s.g. = 1.0)

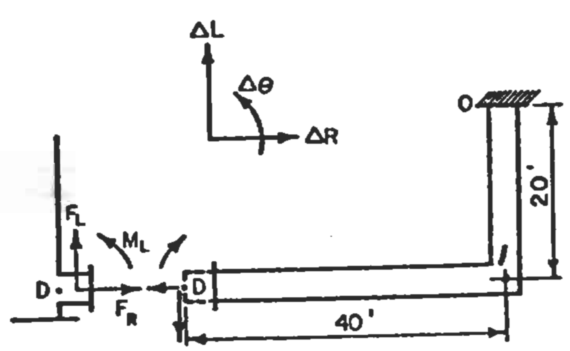


Fig. 6 Flexibility analysis of example problem

Shell thickness = 1.5 in.
Operating temperature = 150°F (80°F above ambient of 70°F).
Modulus of elasticity E at operating temperature = 27.7×10^6 psi
Outward radial growth = ΔR_{OS} = 0.66 in.
Downward rotation = −0.036 rad
Downward rotation and outward growth are due to hydrostatic product head and temperature differential of 80°F; it is assumed that the bottom is restrained from movement by friction forces.

Details of Low-Type Nozzle in Tank:
Nominal pipe diameter = 10-in. schedule 40
O.D. = $2a$ = 10.75 in.
Distance of nozzle centerline from bottom = L = 16 in.
Reinforcing has been provided in shell.

Details of Connected Piping: Refer to Fig. 6 for pipe configuration details and Table 2 for calculations for flexibility coefficients S, S_a, S_{aa}, ..., etc., (obtained using reference [1]). Mean coefficient of thermal expansion = 6.25×10^{-6} in./in. per °F.

$$\Delta R_{OP} = -(l)(\alpha')(\Delta T) = -(40 \times 12)(6.25 \times 10^{-6})(80) = -0.24 \text{ in.}$$

$$\Delta L_{OP} = -(20 \times 12)(6.25 \times 10^{-6})(80) = -0.12 \text{ in.}$$

Step 1: Calculate Stiffness Coefficients for Nozzle-Tank Connection

R/t = 1080/1.5 = 720

a/R = 5.375/1080 = 0.00498 use curves for a/R value of 0.005

$L/2a$ = 16/10.75 = 1.488 use curves for $L/2a$ value of 1.50

From Fig. 11 for radial load:

$$\frac{K_R}{E(2a)} = 9 \times 10^{-4}$$

$$K_R = (9 \times 10^{-4})(27.7 \times 10^6)\,10.75 = 268000 \text{ lb/in.} = 3.216 \times 10^6 \text{ lb/ft}$$

From Fig. 12 for longitudinal moment:

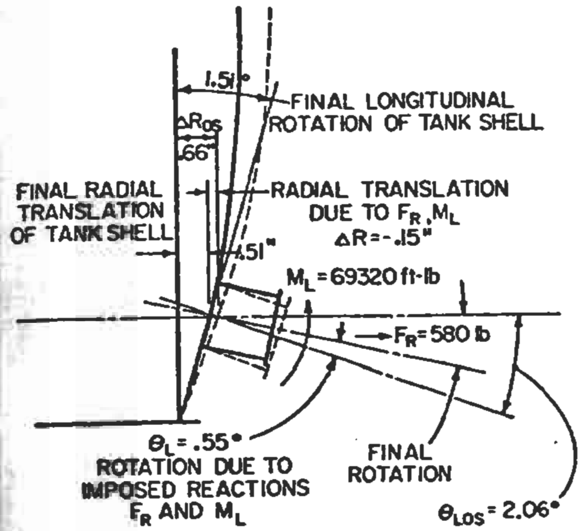
$$\frac{K_L}{E(2a)^3} = 2.8 \times 10^{-3}$$

$$K_L = \frac{(2.8 \times 10^{-3})(27.7 \times 10^6)(10.75)^3}{12} = 7.17 \times 10^6 \text{ ft-lb/rad}$$

$$EI = \frac{27.7 \times 10^6 \times 160.7}{144} = 30.91 \times 10^6 \text{ lb-ft}^2$$

Table 2

MEMBER		D-I	I-O	Σ D-O
ℓ		40.0	20.0	
h		20.0	40.0	
v		0.0	10.0	
$\ell^2/12$		133.3	33.3	
S	ℓ	40.0	20.0	60.0
S_a	(h)(S)	800.0	800.0	1600.0
S_b	(v)(S)	0.0	200.0	200.0
S_{ab}	(v)(S_a)	0.0	8000.0	8000.0
S_{aa}	(h)(S_a)+(S)ℓ^2/12	21332.0	32000.0	53332.0
S_{bb}	(v)(S_b)+(S)ℓ^2/12	0.0	2666.0	2666.0



ig. 7 Final deformation of tank shell-nozzle connection

$$K_R = \frac{30.91 \times 10^6}{3.216 \times 10^6} = 9.61 \text{ ft}^2$$

$$K_L = \frac{30.91 \times 10^6}{7.17 \times 10^6} = 4.31 \text{ ft}$$

$= 7.21 \text{ ft}^2 \qquad \frac{EIL}{K_L} = 5.75 \text{ ft}^2$

rage of $\frac{EI}{LK_R}$ and $\frac{EIL}{K_L} = 6.48 \text{ ft}^2$

te: Average value is used to insure necessary symmetry tiffness matrix.)

tep 2: Solve Compatibility Equations to Determine F_R, M_L. Substituting values in the equations (4), (5), and (6) s:

$$6 + 9.61)F_R - (8000)F_L + (200 - 6.48)M_L$$
$$= 30.91 \times 10^6 \{-0.020 - (0.055)\}$$

$$000)F_R + (53332)F_L - (1600)M_L = 30.91 \times 10^6 (-0.01)$$

$$0 - 6.48)F_R - (1600)F_L + (60 + 4.31)M_L$$
$$= 30.91 \times 10^6 \{-(-0.036)\}$$

ving these linear algebraic equation yields:

$= 580$ lb, $F_L = 2160$ lb, $M_L = 69320$ ft-lb

Step 3: Determine Deflections. The deformations due to calculated loads from the piping are combined with the vements of the nozzle acting with the cylindrical shell due hydrostatic head and temperature differential. These are nmarized in Fig. 7.

Deflection of Tank Shell Due to Imposed Reactions:

flection due to F_R:

$$R = \frac{F_R}{K_R} = \frac{580}{268000} = 0.0022 \text{ in. } (\rightarrow)$$

$$= -\tan^{-1}\left(\frac{\Delta R}{L}\right) = -0.0077 \text{ deg } (\nearrow)$$

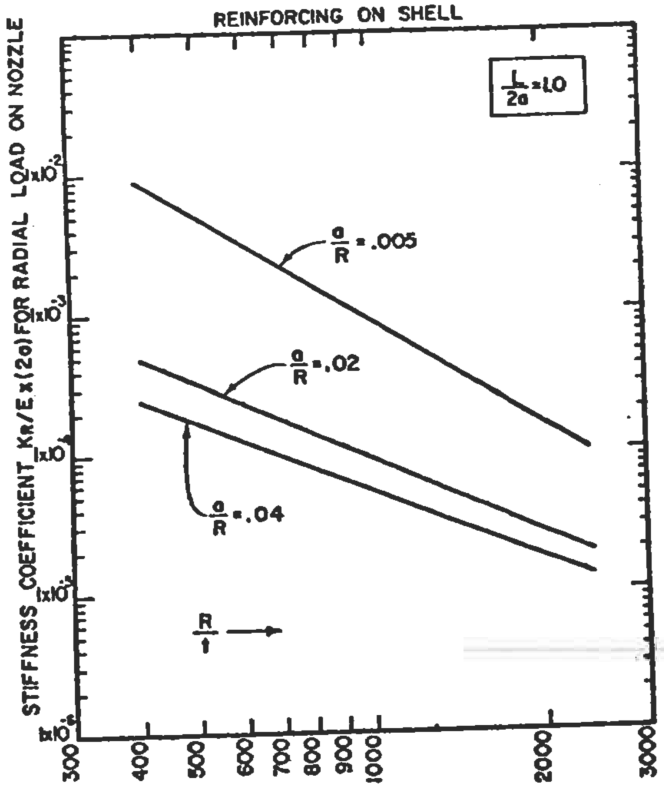


Fig. 8

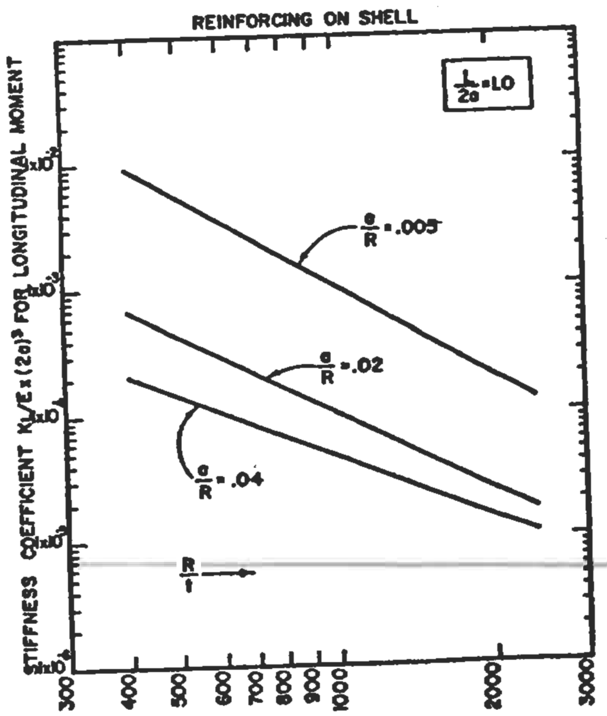


Fig. 9

Deflection due to M_L:

$$\theta_L = \frac{M_L}{K_L} = \frac{69320}{7.17 \times 10^6} \times 57.3 = 0.5540 \text{ deg } (\nwarrow)$$

$$\Delta R = -16 \tan (0.554) = -.1547 \text{ in.} \quad (\leftarrow)$$

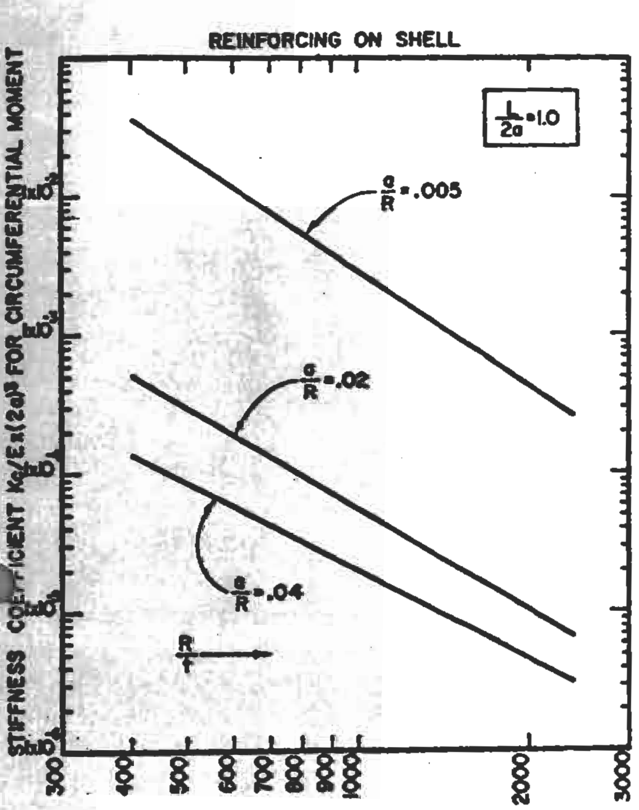


Fig. 10

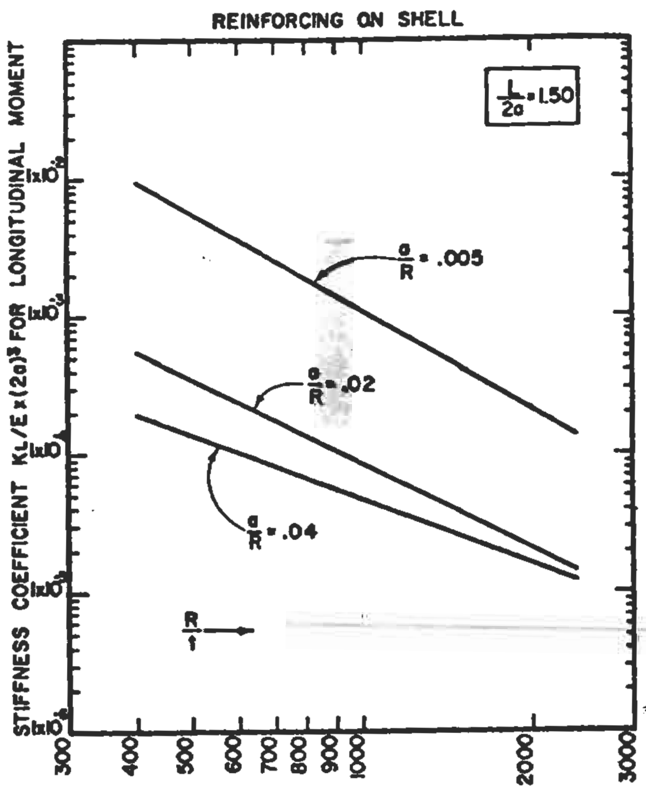


Fig. 12

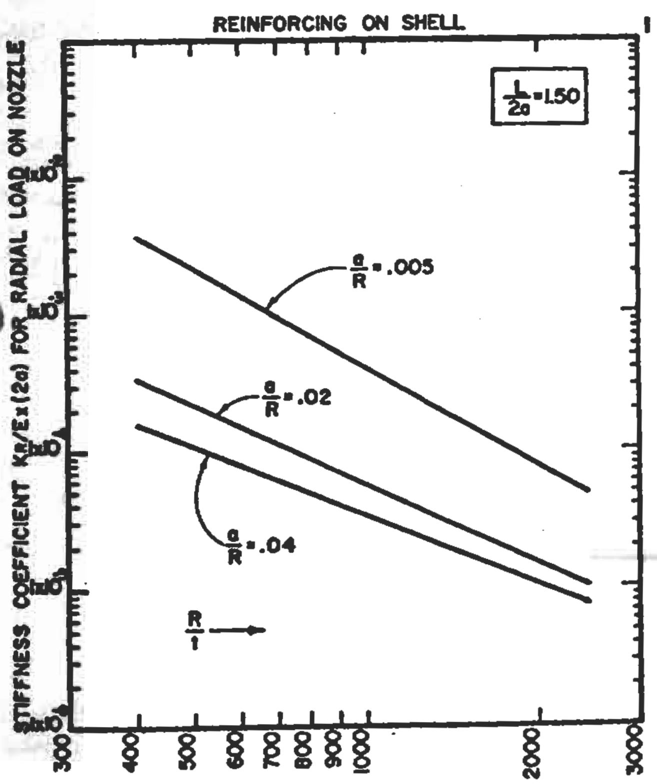


Fig. 11

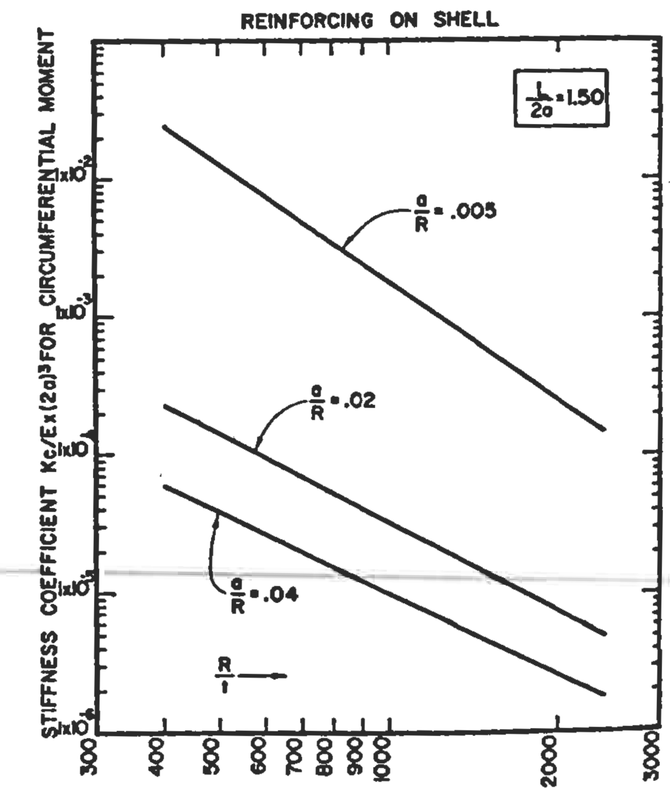


Fig. 13

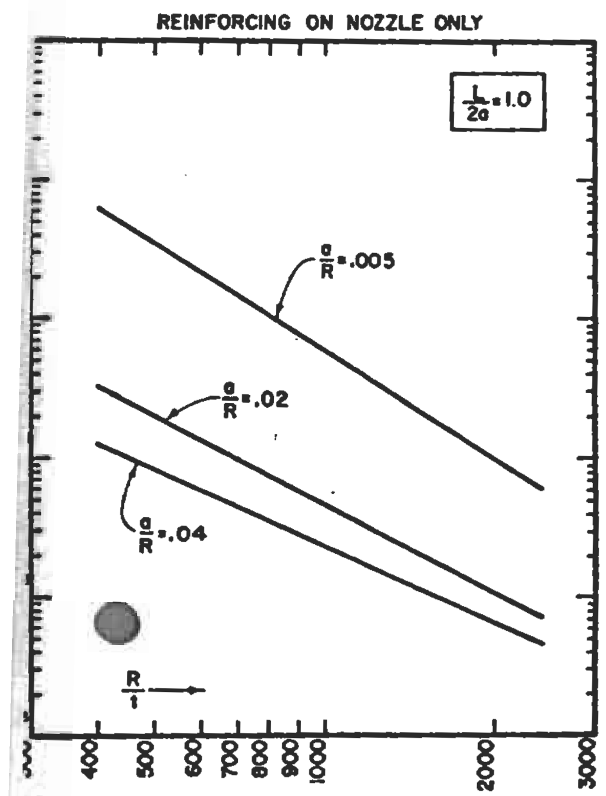


Fig. 14

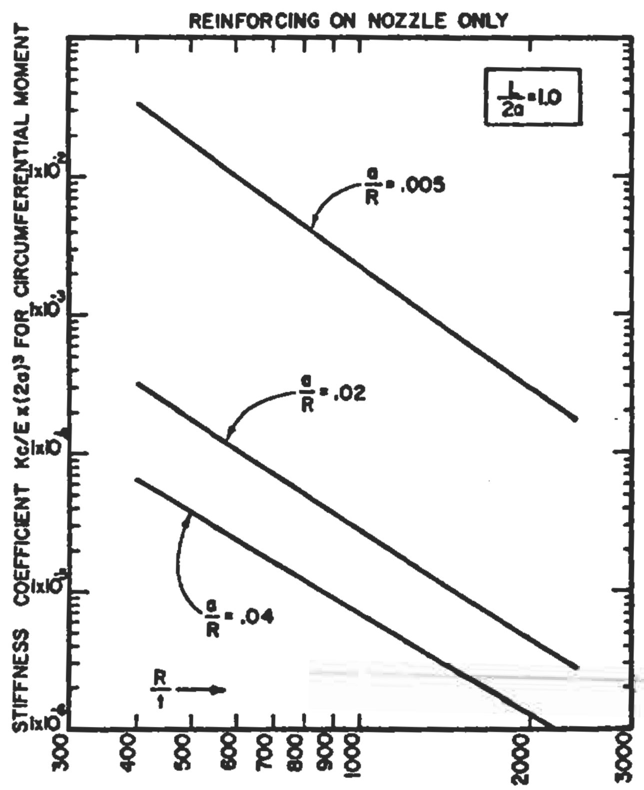


Fig. 16

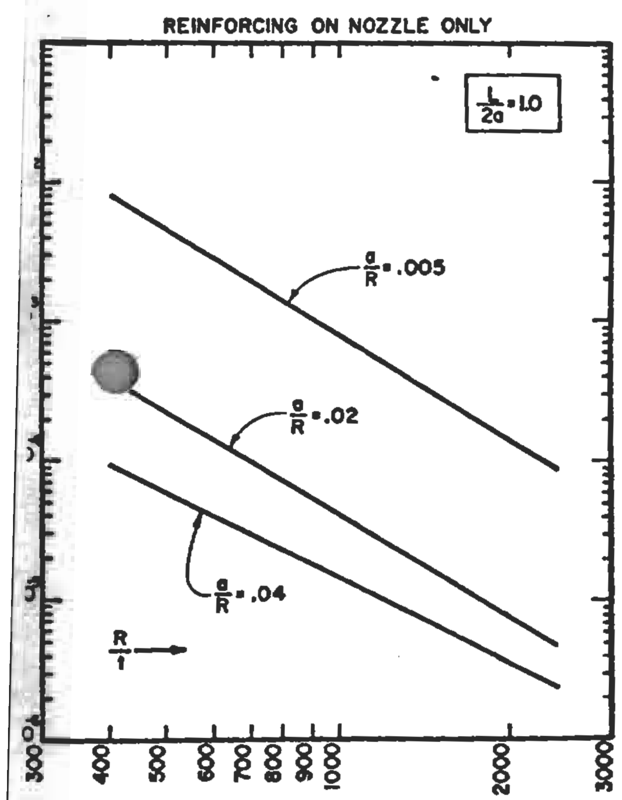


Fig. 15

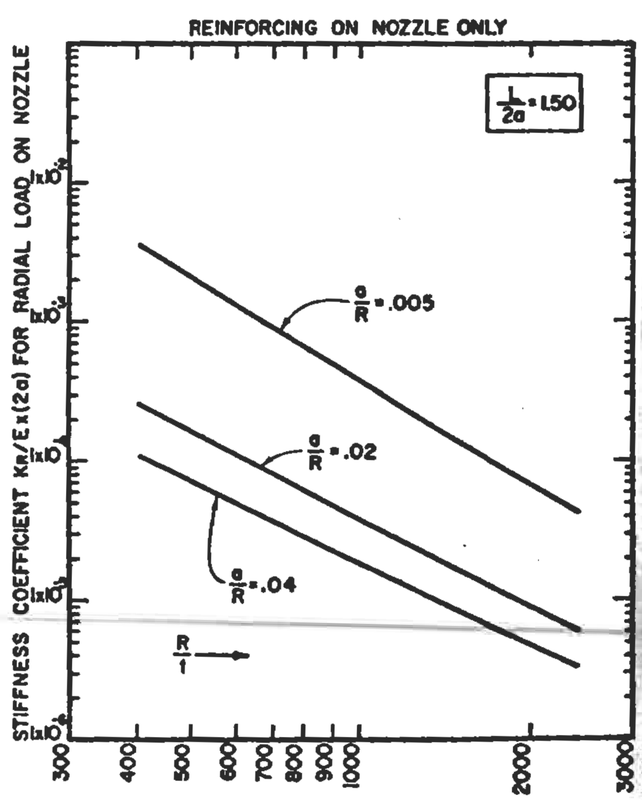


Fig. 17

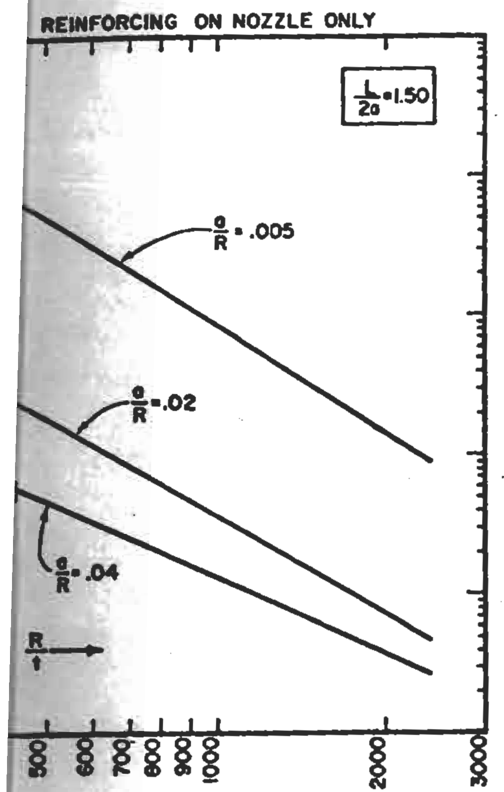


Fig. 18

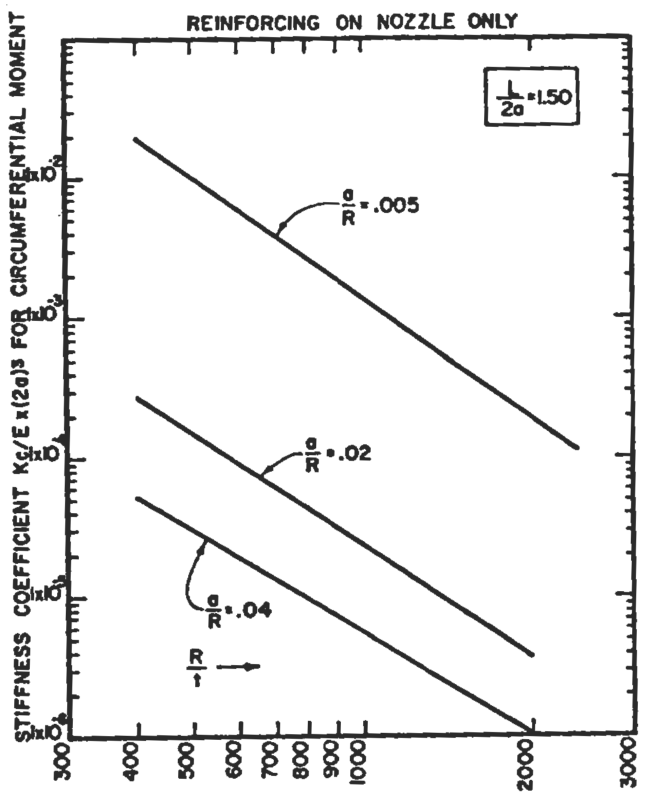


Fig. 19

...n due to both F_R and M_L:

...tion $\Delta R = +0.0022 - 0.1547 = -0.15$ in. (←)

...otation $= \theta_L = -0.0077 + 0.5540 = +0.55$ deg

...nent of tank shell—nozzle connection:

...ement $= \Delta R_{OS} + \Delta R = 0.66 - 0.15 = 0.51$ in.

Longitudinal rotation $= \theta_{LOS} + \theta_L = -2.06 + 0.55 = -1.51$ deg (↗)

Check Acceptability of Calculated Loads From Example Problem Using Suggested Criteria

$\sqrt{Rt} = \sqrt{1080(1.5)} = 40.25$

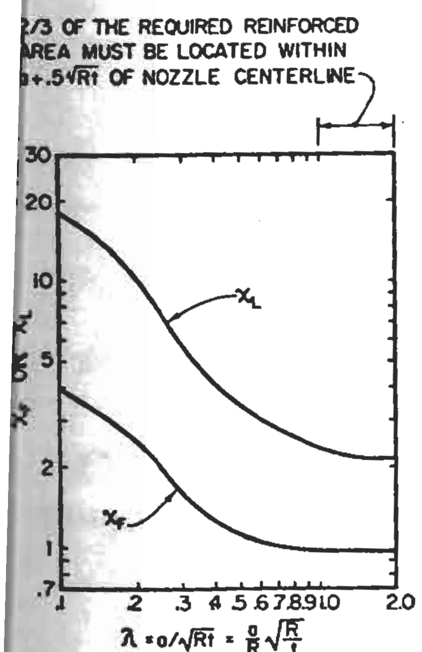


Fig. 20

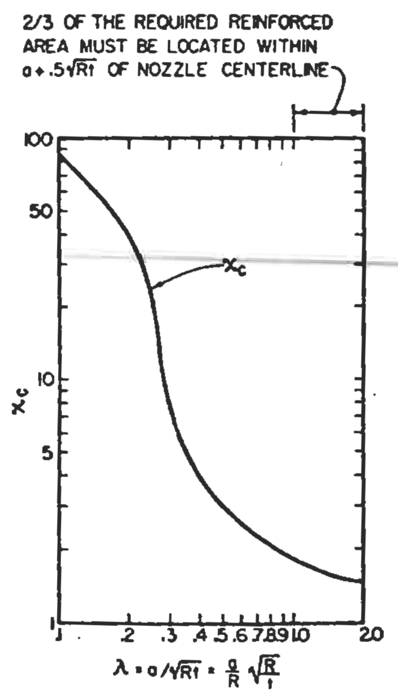


Fig. 21

$\frac{.375}{0.25} = 0.13$

$\frac{62.4}{144}(70 - 1.33)\pi(5.375)^2 = 2700$ lb

$= 3.4; \chi_L = 15.0$ (refer to Fig. 20)

5.375)/40.25 = 0.53

5.375)/40.25 = 0.26

. does not apply for sample problem, $M_C = 0$

orces at nozzle-shell connection are:

$M_L = 832000$ in.-lb

$\left(\frac{F_R}{F_P}\right) = \frac{0.13}{(2)(3.4)}\left(\frac{580}{2700}\right) = 0.004$

$$\frac{\lambda}{\alpha\chi_L}\left(\frac{M_L}{F_P}\right) = \frac{0.13}{(5.38)(15)}\left(\frac{832000}{2700}\right) = 0.5$$

The nomograph shown in Fig. 5(a) has been drawn for this nozzle configuration. The point (0.004, 0.5) lies within the boundaries; therefore, the calculated nozzle loads are acceptable for this nozzle-tank configuration.

Check Stresses in Nozzle Neck. (Assume nozzle wall thickness = 0.5.)

$$S = \frac{F_P}{2\pi at} + \frac{F_R}{A} + \frac{(M_L)(c)}{I}$$

$$= \frac{2700}{2 \times \pi \times 5.4 \times .5} + \frac{580}{16.6} + \frac{832000 \times 5.4}{212.0}$$

$$= 159 + 36 + 21192$$

$$= 21387 \text{ psi}$$